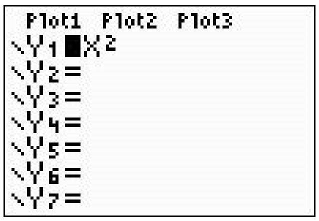
Notes for 3.1 Quadratic Functions and Models
(pp. 294 - 303)

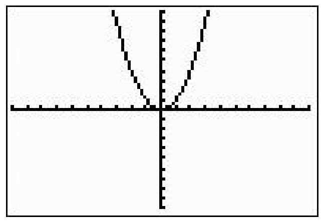
Name:
Date:
Instructor:

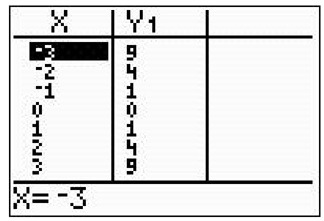
Topics: Quadratic Functions and Their Characteristics, Graphing, and Modeling

A function is a **quadratic function** if $f(x) = ax^2 + bx + c$, where *a, b,* and *c* are real numbers with $a \neq 0$.
* This is the *algebraic* form of the function.

I. Graphing Techniques (pp. 295 – 296, 299 – 300)
We begin with the basic quadratic function: Enter $Y_1 = x^2$



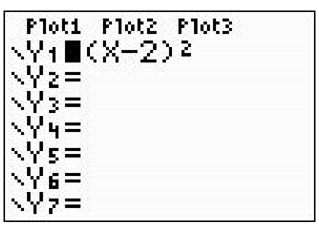


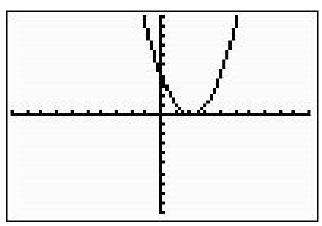


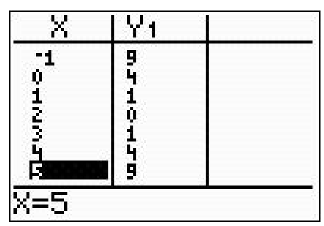
The axis of symmetry is *x* = 0, since that is the line of reflection from the left side of the graph to the right side of the graph. Notice the symmetry of the points in the TABLE, also.
The domain is $(-\infty, \infty)$ and the range is $[0, \infty)$. (Mistake on the video graph.)

The quadratic function can also be written in the form $f(x) = a(x-h)^2 + k$, where *h* is the horizontal translation (left or right) and *k* is the vertical translation (up or down).

Ex. Graph $f(x) = (x-2)^2$. State its domain and its range.

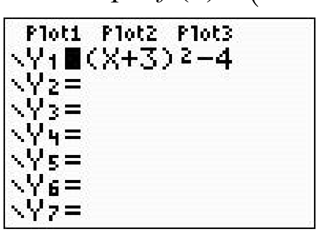


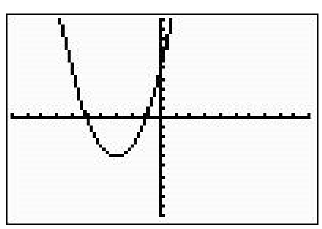


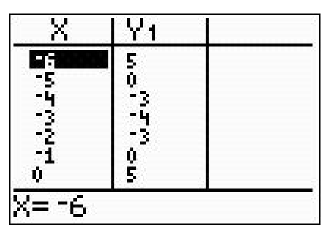


The domain is $(-\infty, \infty)$ and the range is $[0, \infty)$.

Ex. Graph $f(x) = (x+3)^2 - 4$. State its domain and its range.







II. Completing the Square (Omit pp. 297 – 298)
We omit the completing the square technique here. Again, there are better ways to accomplish the same result.

III. Graphing a Quadratic Using the Algebraic Form of the Function (The Vertex Formula) (pp. 299 – 300)

Recall: The quadratic function can be written as $f(x) = ax^2 + bx + c$ (algebraic form) and also $f(x) = a\left(x-h\right)^2 + k$ (graphing form). The connection between the two forms is made by the vertex (*h, k*), where $h = \frac{-b}{2a}$ and $k = f(h)$ (which is found by substituting the h value into the quadratic.

Characteristics of a Quadratic Function:
1. The graph is a _________ with the vertex *(h, k)* and the vertical line *x = h* as its axis of symmetry.
2. It opens **up** if $a > 0$ (has a *positive* leading coefficient), or
   it opens **down** if $a < 0$ (has a *negative* leading coefficient).
3. It is **broader** than the basic graph of x^2 if $|a| < 1$ (is a proper fraction), or
   it is **narrower** than the basic graph of x^2 if $|a| > 1$ (is an improper fraction or whole number).
4. The **y-intercept** is $f(0) = c$ and is written as (0, *c*).
5. The **x-intercept(s)** are found from the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ if } b^2 - 4ac \ge 0 \text{ (is positive or 0)}$$

If $b^2 - 4ac < 0$ (is negative), then there are no x intercepts. (The graph floats above the x axis or remains below the x axis, so that it doesn't cross there).

Ex. Find the axis of symmetry and the vertex of the parabola having the equation
$f(x) = 2x^2 + 4x + 5$.

First, find the vertex:

$$x = \frac{-b}{2a} =$$

Now, the *y* coordinate is found by substituting and finding *f (-1)*:

$y = f(-1) = 2(-1)^2 + 4(-1) + 5 =$

*Note that finding *y* is easily done by using the STO→feature of the calculator.

II. Using Quadratics to Model Data

Ex. page 300, Example 5 a – f
a. Video
b. The calculator WINDOW at the top sets the dimensions of the GRAPH screen. You can adjust the dimensions using WINDOW: XMIN is the left side of the x axis; XMAX is the right side of the x axis; XSCL is the scale that the calculator uses to write the hashmarks of the number line; YMIN sets the depth of the y axis; YMAX sets the height of the y axis; YSCL sets the distance between hashmarks on the y axis number line.
[XMIN, XMAX, XSCL x (by) YMIN, YMAX, YSCL] is how some texts denote this information.
* ZOOM 6:Standard resets the WINDOW to [-10, 10, 1 x –10, 10, 1]
c. Video

d. "Finding the maximum or minimum of a parabola" is really finding the vertex coordinates. The time is the x coordinate and the height that is reached is the y coordinate.

There are 2 ways to do this:
1. The calculator way shown on the video—Equation into Y_1, then 2^{nd} CALC (on top of TRACE) 4:Maximum (or 3:Minimum), then move cursor a little to the left side of what looks like the top of the parabola, ENTER, then a little to the right side of the top, ENTER, then always ENTER to make the calculator guess (the calculator guesses really well!!).
The drawback to this is that the calculator has to "see" the maximum area in its view screen, so it may take an adjustment of the WINDOW to have that happen.
2. The $\frac{-b}{2a}$ way: Find the x coordinate, then substitute it back in to get the y coordinate that goes with it. This method works, even when the equation coefficients are big numbers.

e. "What time will the object be *higher than 160ft. high*?" means that the quadratic will need to be > only.
There is a mistake in the video with the sign of the 5… it's supposed to go like this:

$-16t^2+80t+100>106$ (Now, make inequality compare with 0)

$-16t^2+80t-60>0$ (Now, reduce the inequality by dividing by –4—always good to reduce to have smaller numbers to work with…

$4t^2-20t+15<0$ (Careful of the signs and switching the inequality)

$x=\frac{20\pm\sqrt{20^2-4\bullet4\bullet15}}{2\bullet4}$ (Careful when evaluating the quadratic formula—I advise students *not* to rely entirely on the calculator, but rather enter to get the discriminant first--)

$x=\frac{20\pm\sqrt{160}}{8}$ Now, since you need a single value, rather than the exact answer with the square root, you'll need to put the $20+\sqrt{160}$ in the calculator, ENTER, then ÷ 8. Round to the appropriate decimal place. Now, reenter the numerator as 20 - $\sqrt{160}$, ENTER, and divide by 8 to get the other possible value for *x*.
The two values that you get break the number line into 3 parts, with the *x* values as the critical values. Since the problem was > (greater than), the solution needs to be in the "wings" form of $(-\infty,.92)\cup(4.08,\infty)$
f. "What time will the object *hit the ground*?" means that the equation will always = 0 (that's the elevation of the ground). Use the quadratic formula again, with the help of the calculator. Time will always be the positive value, so that's the answer.

Assignments:
Text: pp. 303 – 310 #1, 3, 5 – 8, 12, 13 – 25 odd, 27 – 30, 41 – 46, 57, 61, 63, 65c,e,f with calculator regression (as in Section 2.4)