

Notes for 3.4 Polynomial Functions: Graphs, Applications, and Models (pp. 331 – 342)

Name:
Date:
Instructor:

Topics: Graphs of monomial function  $ax^n$ , general polynomial functions, Turning Points, End Behavior, Real Zeros, Models and Curve Fitting

I. Graphing Basic Monomial Functions (pp. 331 – 332)

All polynomial functions are continuous.

If the function has an **odd** exponent (degree), then the graph looks like a tall snake.

Ex.  $f(x) = x^3$  and  $f(x) = x^5$

If the function has an **even** exponent (degree), then the graph looks like a smiley or a frowny.

Ex.  $f(x) = x^4$  and  $f(x) = -x^6$  (Recall that the negative leading coefficient reflects the graph across the  $x$  axis.)

Summary of the Behavior of Monomial Graphs: The shape of  $f(x) = ax^n$

$n$	$a$	Shape	Example
even	positive	↑ ↑	$f(x) = x^2$ smiley
even	negative	↓ ↓	$f(x) = -x^2$ frowny
odd	positive	↓ ↑	$f(x) = x^3$ snakes from 3 <sup>rd</sup> into 1 <sup>st</sup> quadrant
odd	negative	↑ ↓	$f(x) = -x^3$ snakes from 2 <sup>nd</sup> into 4 <sup>th</sup> quadrant

Recall: Translations (pp. 332 – 333)

1. Vertical and Horizontal Translations of  $f(x) = a(x - h)^2 + k$

a. Vertical Translation:

The graph is shifted  $k$  units *up* if  $k > 0$  (positive), or  $|k|$  units *down* if  $k < 0$  (negative)

b. Horizontal Translation:

The graph is shifted  $h$  units to the *right* if  $h > 0$  (subtraction sign is in the parentheses), or  $|h|$  units to the *left* if  $h < 0$  (addition sign is in the parentheses).

2. The coefficient makes the graph **narrower** if  $|a| > 1$  (an improper fraction or a whole number, or **wider** if  $0 < |a| < 1$  (a proper fraction).

3. A negative leading coefficient reflects the graph about the  $x$  axis.

Ex. Describe the translations of  $g(x) = -(x - 2)^3 - 3$  in a sentence.

The basic function of  $f(x) = x^3$  has been reflected across the  $x$  axis, been shifted 2 units to the right, and shifted 3 units downward.

## II. Zeros of a Function (pp. 333 – 334)

A **zero** of a function = solution when the equation is  $0 =$  an **x intercept**.

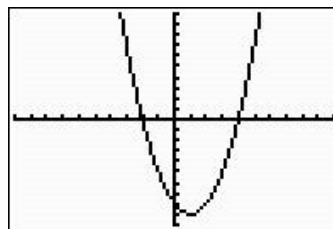
Ex. For  $g(x) = x^2 - 2x - 8$   
 $= (x + 2)(x - 4)$

Find the **zeros** of  $g(x)$ , we solve  $g(x) = 0$ .

$$\begin{aligned}x^2 - 2x - 8 &= 0 \\(x + 2)(x - 4) &= 0 \\x + 2 = 0 \text{ or } x - 4 &= 0 \\x = -2 \text{ or } x &= 4\end{aligned}$$

The **solutions** of  $x^2 - 2x - 8 = 0$  are  $-2$  and  $4$ .

The zeros of  $g(x)$  are the x-coordinates of the **x-intercepts** of the graph of  $y = g(x)$  at  $(-2, 0)$  and  $(4, 0)$ .



## III. Multiplicity (Section 3.3, p. 324)

The *Fundamental Theorem of Algebra* states that every polynomial function of degree 1 or more has at least one complex (real or imaginary) zeros. (Blue box)

The *Number of Zeros Theorem* states that a polynomial function of degree  $n$  has at most  $n$  distinct zeros.

These two theorems give us the range of how many solutions/x intercepts/zeros a polynomial function can have.

The **multiplicity** of a zero (solution) has to do with the occurrences of the zeros. If a factor occurs once, then it has a multiplicity of 1; if it occurs twice, then it has a multiplicity of 2, etc.

That multiplicity gives another connection between the zeros and the graphs:

If a factor of the polynomial has an **odd degree** multiplicity, then the graph **crosses** the x axis at the zero value.

If the factor of the polynomial has an **even degree** multiplicity, then the graph **touches** the x axis at the zero value.

Ex. State the zeros, their respective multiplicities, and whether the graph crosses or touches the x axis at that zero, for the function  $f(x) = .05(x - 4)^3(x + 2)^2$

The zeros are **4**, with the **multiplicity of 3**, and **-2**, with a **multiplicity of 2**.

The graph **crosses the x-axis at 4**, since the degree of that factor is **odd**.

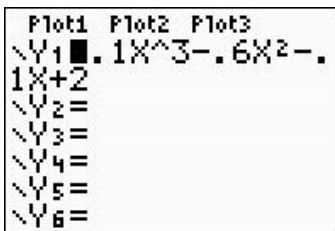
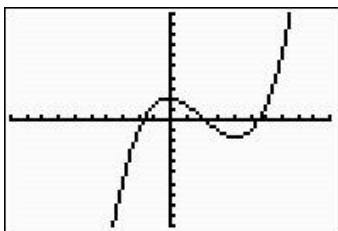
The graph **touches the x-axis at -2**, since the degree of that factor is **even**.

Ex. Find the real zeros of the function  $f(x) = 0.1x^3 - 0.6x^2 - 0.1x + 2$ . Approximate the zeros to three decimal places.

Enter the function into  $Y_1$ , then ZOOM 6 to get started. The graph shows 3 x-intercepts, which is all that is possible, according to the Number of Zeros Theorem. Use the calculator to find the coordinate names of each x-intercept, using 2<sup>nd</sup> CALC 2:ZERO. Move the cursor a little to the left of the intercept, ENTER, then move the cursor a little to the right down into the 4<sup>th</sup> quadrant, ENTER, then ENTER to make the calculator guess. It will find the x-intercept of (2.154, 0). To find another intercept, use 2<sup>nd</sup> CALC 2:ZERO again. The cursor will be where it left it at the 2.154, so move the cursor near the left side of either the right intercept or the left one, whichever

you prefer to find next. Mark the left and the right boundaries as you did before, then ENTER to make the calculator guess again. It should return  $x = -1.680$  on the left and  $x = 5.526$  on the right.

All three of these zeros cross the x axis, as shown on the graph.



### III. Turning Points and End Behavior (pp. 334 – 335)

A **turning point** on a graph is where the graph changes from increasing to \_\_\_\_\_ or from decreasing to \_\_\_\_\_. A polynomial function with a degree of  $n$  has **at most  $n - 1$**  turning points.

These turning points are found using the calculator 2<sup>nd</sup> CALC 3:MINIMUM or 4:MAXIMUM with the *left ENTER- right ENTER – ENTER* to get the coordinates of the point.

The **end behavior** of the graph of a polynomial function is the same as the behavior of the monomial function (chart, page 1 of Notes).

Ex.  $f(x) = -0.5x^4 + 0.25x - 5$  has at most 3 turning points. The end behavior of  $f(x)$  is  $\downarrow \downarrow$ .

Ex.  $g(x) = 2x + x^3 - x^5$  has at most 4 turning points. The end behavior of  $g(x)$  is  $\uparrow \downarrow$ .

### IV. Sketching the Graph of a Polynomial

Ex. Graph  $f(x) = (x - 4)(x + 3)(2x - 1)$

1. Enter the function into  $Y_1$  and ZOOM6 to get an idea of the intercepts.
2. Adjust the WINDOW to see the complete graph. Let the TABLE help determine the dimensions of the graphing window. Can you see from the  $Y_1$  values that you need to adjust the Y dimensions to go deep enough to see  $-30$  clearly? Set the window at  $[-10, 10, 1]$  by  $[-40, 40, 10]$ ... no change in the X dimensions, but give the Y axis some room, and reset the hashmarks to 10 so the calculator will show them clearly, instead of a bold-face axis. Label your sketch likewise.
1. Plot the zeros of 4, -3, and  $\frac{1}{2}$  on your sketch.
2. Plot the y-intercept from the TABLE (0, \_\_\_) (estimate)
3. Find the turning points using 2<sup>nd</sup> CALC MAX or MIN
4. Sketch the graph using the info about the end behavior of a polynomial with a degree of 3 and a positive leading coefficient (positive 1<sup>st</sup> parentheses).
5. Use the TABLE to get more points if needed.

Omit the video portion from 9:09 – 15:01. It's easier to do the sketch like we did above.

Ex. Graph  $f(x) = 18x^4 - 87x^3 + 56x^2 + 20x - 16$ . Find the real zeros using 2<sup>nd</sup> CALC 2 and omit the video portion of 15:12 – 17:25.

\*Omit The Intermediate Value and Boundedness Theorem sections of the text, p. 338 – 340.

V. Models and Curve Fitting (pp.341 – 342)

Ex. The number of lung cancer cases per 100,000 females in the year  $t$  (where  $t = 0$  corresponds to 1930) can be modeled using the function  $f(t) = .00028t^3 - .011t^2 + .23t + .93$ .

a) Use the calculator to graph  $f(t)$  for  $0 \leq t \leq 60$

b) Determine the years where  $f(t)$  exceeds 30.

a) Set WINDOW using the domain set by the problem, which is from 0 to 60 and the YMIN/YMAX to see the complete graph.

b) Enter  $Y_2$  as 30. Use 2<sup>nd</sup> CALC 5:INTERSECTION ENTER ENTER ENTER to get the calculator to find the point of intersection of the two functions. The x-coordinate is the solution, which has to be added to the beginning year of 1930.

VI. Another graphing problem

Ex. Graph  $f(x) = x^5 - 2x^4 - 3x^3 - 4x + 10$  (Omit Boundedness and the tests part 21:57 – 23:03)

Find the real intercepts and the turning points (a maximum between  $-1.71$  and  $1.12$  and a minimum between  $2.4$  and  $-27.8$ )

Assignments:

Text: pp. 342 – 348 #1– 8, 9, 13, 21 – 27 odd, 29 – 39 odd graph and name x-intercepts, 63, 65, 71 – 77 odd, 98 b, c, d (regression like Section 2.4)