

## Notes for 3.6 Variation (pp. 368 – 372)

Topics: Direct, Indirect, and Joint Variation

Name:
Date:
Instructor:

When a circumstance pairs numbers with a *ratio* that is constant, then there is a **direct variation**. That ratio is called the **constant of proportionality**.

These problems always fall into a pattern: there is one sentence that has a complete fact relating the values of the story, and there is one sentence with incomplete information that you must complete. This sets up the order of solution to a problem link this: 1. Find the constant of proportionality ( $k$ ). 2. Write a general formula using  $k$  value and the variables of the problem. 3. Substitute the values from the incomplete statement and answer the question.

### I. Direct Variation (pp. 369 – 370)

**Direct Variation** can be stated as: “ $y$  varies \_\_\_\_\_ as  $x$ ” or “ $y$  is directly proportional to  $x$ ”. This means that there is a nonzero real number,  $k$ , such that  $y = k * x$ . The **constant of proportionality** is \_\_\_\_\_ .

Ex. The total cost of making copies varies directly as the number of copies made. If the cost of making 520 copies is \$20.80, what is the cost of 178 copies?

The first sentence sets up the type of variation of the problem:  $C = k w$ , where  $C$  is the cost and  $w$  is the number of copies.

Find the  $k$  for the problem by substituting the complete information:  $20.80 = k \cdot 520$   
 $k =$

Now write the formula using the value of  $k$ :  
and substitute the partial information

Variations can happen in other situation besides linear:

Ex.  $y$  varies directly as the cube of  $x =$

Ex.  $y$  varies directly as the  $n^{\text{th}}$  power of  $x =$

\* Note the units of each value in the formula (\$, ft, inches, hrs, etc) as a hint of where that value needs to be substituted.

### II. Inverse Variation (pp. 370 – 371)

**Inverse Variation** is a situation where the *product* is constant between two numbers.

This can be stated as “ $y$  varies inversely as the  $n^{\text{th}}$  power of  $x$ ” or  $y$  is inversely proportional to the  $n^{\text{th}}$  power of  $x$ ”. This means that there is a nonzero real number,  $k$ , such that  $y = \frac{k}{x^n}$ .

Ex. The illumination produced by a light source varies inversely as the square of the distance from the source. The illumination of a source at 5 m is 70 candela. What would it be at 12 m?

Again, the first sentence sets up the relationship, the second is the complete information, and the third is the incomplete with the question.

1. Write the relationship equation.
2. Substitute the values from the complete statement to find  $k$ .
3. Use  $k$  and the original relationship to write a general formula. \_\_\_\_\_
4. Use the formula with the incomplete information to solve the problem.

\*Remember that multiplication undoes division.

\*Note that this is a situation that can be cross-multiplied, if necessary.

Techniques of Solving Variation Problems:

1. Write the general relationship among the variables as an equation. Use the constant  $k$ .
2. Substitute the given values of the variables and find the value of  $k$  (complete info).
3. Substitute this value of  $k$  into the equation to write a general formula.
4. Use this formula solve for the required unknown.

III. Joint and Combined Variation (pp. 371 – 372)

**Joint Variation** relates more than one factor with the constant of variation,  $k$ . This is sometimes called multi-variable variation. Use the reciprocal, if needed, to isolate  $k$ .

Ex. The volume ( $V$ ) of a certain box varies jointly as its width ( $w$ ) and its height ( $h$ ). If the volume is  $27 \text{ ft}^3$  when the width is 3 ft. and the height is 4.5 ft., what is the volume of a width of 1.5 ft. and a height of 7.5 ft.?

Sometimes the joint variation relates both direct and indirect variations, called Combined Variation:

Ex. Suppose  $c$  varies directly as the square of  $a$  and inversely as the square root of  $b$ . If  $c = 60$  when  $a = 2$  and  $b = 9$ , find  $c$  if  $a = 3$  and  $b = 16$ .

\*Remember that the direct (one) or joint (more than one) values go into the *numerator*, and the inverse values go into the *denominator* of the formula.

Assignment:

pp. 372 – 374 #11 – 21 odd, 25, 29