

## Notes for 4.1 Inverse Functions (pp. 390 – 398)

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| Name:       |
| Date:       |
| Instructor: |

Topics: One-to-One Functions, Inverse Functions

### I. Inverse Functions and One – to – One Functions (pp.390 – 392)

Recall:

Inverses of arithmetic are addition and subtraction, multiplication and division, etc.

A function takes a value of the domain,  $x$ , and operates on it using the function formula, then gives you an answer that is in the range,  $y$ .

To find an inverse of the function, we want to “undo” the function and have the range value,  $y$ , give back its answer,  $x$ , in the domain.

Ex.  $f(x) = 5x$  and  $g(x) = \frac{x}{5}$

For  $f(6) = 5 * 6 = 30$  and  $g(30) = \frac{30}{5} = 6$ . These two functions are inverses of each other.

How to determine whether a function is one-to-one: (p. 391)

1. Work several examples and see if one  $y$  value is determined by more than one  $x$  value.

Ex.. Determine whether the function defined by  $f(x) = \sqrt{25 - x^2}$  is one-to-one.

Substituting 3 and  $-3$

$f(3) =$  \_\_\_\_\_ and  $f(-3) =$  \_\_\_\_\_

Since these answers are the same positive 4, then this function is not 1 – 1.

2. Use the Horizontal Line Test on the graph. This is very similar to the Vertical Line Test that determines whether the graph is a function or not. It states that any horizontal line passes through the graph in only one place at any part of the graph.

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### II. Finding the Inverse Function (pp. 394 – 397)

If a function is a one – to – one function, then it has an inverse.

To find the inverse of a function, you: (p. 394)

1. Write the function using the variable  $y$  instead of the  $f(x)$  label.
2. Interchange the  $x$  and  $y$  in the equation. (This will do it generically for all the values of  $x$  and  $y$ ) instead of you having to do them one at a time from the TABLE.)
3. Solve for the new  $y$ .
4. Write the answer using the  $f^{-1}(x)$  notation.

Ex.  $f(x) = x^3 + 1$

$$y = x^3 + 1$$

$$x = y^3 + 1$$

(replace the  $f(x)$  with  $y$ )

(interchange the  $x$  and  $y$ )

(solve for the new  $y$ )

so  $f^{-1}(x) =$

Ex. Write the equation for the inverse of  $f(x)$  in the form of  $f^{-1}(x)$  for

$$f(x) = 3x - 4$$

III. The graph of the inverse is a reflection of the original function round the line  $y = x$ .

IV. Summary of Important Facts about Inverses: (p. 397)

1. If  $f$  is 1-1, the  $f^{-1}$  exists.

2. The domain of  $f$  is equal to the range of  $f^{-1}$ , and the range of  $f$  is equal to the domain of  $f^{-1}$ .

(This is because you interchange all the  $x$  and  $y$  values.)

3. If a point  $(a, b)$  lies on the graph of  $f$ , then  $(b, a)$  lies on the graph of  $f^{-1}$ , so the graphs of  $f$  and  $f^{-1}$  are reflections of each other across the line  $y = x$ .

4. To find the equation for  $f^{-1}$ , replace  $f(x)$  with  $y$ , interchange  $x$  and  $y$ , and solve for the new  $y$ .

This gives you the inverse.

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Assignment:

Text: pp. 398 – 401 #3 – 25 odd, 33 – 51 odd, 53a, 57a, 59a, 63 – 73 odd