Notes for 4.3 Logarithmic Functions (pp. 418 – 427)

Name:
Date:
Instructor:

Topics: Definition of Log, Log Equations, Log Functions, Properties of Logs (This video picks up in the middle of the section, so I've included some basic information to build on.).

I. Logarithms

Def. For all real numbers *y*, and all *positive* numbers *a* and *x*, where $a \neq 1$:

$y = \log_a x$ (log form) if and only if $x = a^y$ (exponential form).

The log is the *inverse* of the exponential and is the **logarithm with base *a*.** *These are two different ways to say the same thing… one in exponential form and one in logarithmic form.
*Note that both values that are with the "log", the *a* (base) and the *x* (which is called the argument) *must* be positive.

II. Converting between exponential and log form (p. 419, chart) (Learn this pattern format! Doing this conversion is a tool that is applied to several different areas of solving equations and real-world applications.)

Ex. If $\log_2 8 = 3$, then $2^3 = 8$ and if $2^3 = 8$, then $\log_2 8 = 3$.

Ex. If $2^5 = 32$, then $\log_2 32 = 5$ and if $\log_2 32 = 5$, then $2^5 = 32$.

Ex. If $3^5 = 243$, then ______________ and if _____________, then $3^5 = 243$.

Ex. If $\log_{\frac{1}{2}} 8 = -3$, then _____________ and if ___________, then $\log_{\frac{1}{2}} 8 = -3$.

Ex. If $10^0 = 1$, then _____________ and if _________________, then $10^0 = 1$.

Ex. If $\log_{7.5} 7.5 = 1$, then ____________ and if ______________, then $\log_{7.5} = 7.5$

*Get more practice from "Becoming Friends With Logarithms", parts I, II)

III. Solving One Type of Log Equations (p. 419 – 420)

There are 2 types of equations with logs in them:

a. Equations with logs in **some** of the terms (here and Section 4.4)

b. Equations with logs in **all** terms. (Section 4.4)

First we look at equations with logs in some (or only one) term(s).

Ex. $\log_a\left(\frac{4}{49}\right) = 2$ (*Use the definition to convert it to an exponential form.)

Ex. $\log_8 x = \frac{4}{3}$ (*Use the definition of log and use the calculator with ( ) around $\frac{4}{3}$ to get the answer. Omit the change to the radical… use the calculator instead)

IV. Logarithmic Functions (p. 420 – 422)

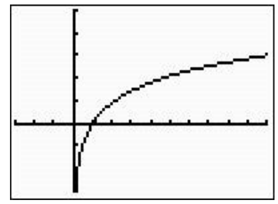
Def. If $a > 0$ (is positive), $a \neq 1$, and $x > 0$ (is positive), then

$f(x) = \log_a x$ defines the ________________ ________________ with base ____.

*Note that the base must be _________ and different from 1 and the argument x must be ________________ as well.

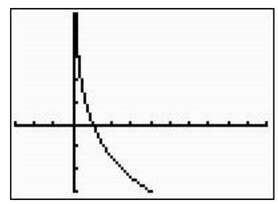
Characteristics of the graph of a logarithmic function: (2 forms of $f(x) = \log_a x$) (p. 421)

A. If $a > 1$, then: (Ex. $y = 3^x$)



a. The graph reflects about the line $y = x$.
b. The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.
c. The y axis is the vertical asymptote and there is no y-intercept.
d. The x-intercept is the point (1, 0) for all graphs where $a > 1$.
e. The graph contains the points (1, 0), (a, 1) and $\left(\frac{1}{a}, -1\right)$.
f. The graph is **increasing.**

B. If $0 < a < 1$, then: (Ex. $y = \frac{1}{3}^x$)



a. The graph reflects about the line $y = x$.
b. The domain is $(0, \infty)$ and the range is $(-\infty, \infty)$.
c. The y axis is the vertical asymptote and there is no y-intercept.
d. The x-intercept is the point (1, 0) for all graphs where $a > 1$.
e. The graph contains the points (1, 0), (a, 1) and $\left(\frac{1}{a}, -1\right)$.
f. The graph is **decreasing.**

V. Translations still work (omit p. 423, Ex 3) (Omit video 10:08 – 11:20)

VI. Properties of the Logarithmic Function (pp. 423 – 426)

*These work because logarithms **are** exponents. Recall all the exponent rules.

1. Product: $\log_a xy = \log_a x + \log_a y$ (since $a^x \bullet a^y = a^{x+y}$).
2. Quotient: $\log_a \frac{x}{y} = \frac{\log_a x}{\log_a y}$ (since $\frac{a^x}{a^y} = a^{x-y}$).
3. Power: $\log_a x^r = r \log_a x$ (since $(a^r)^x = a^{rx}$)
4. $\log_a a = 1$ (since $a^1 = a$)
5. $\log_a 1 = 0$ (since $a^0 = 1$)

These are used to expand and contract (condense) log statements. Expansions must convert any radical to its exponential form, then move the powers, then expand the quotients, then expand the parts of the products.

Ex. Write $\log_p \sqrt[3]{\frac{m^5 n^4}{t^2}}$ as a sum, difference, and/or product. (Remember that $\sqrt[n]{a^m} = a^{\frac{m}{n}}$)

We can use these rules to condense an expression to a single log statement by using them in the opposite direction.

Ex. Write $\log_a x + 2\log_a y - 3\log_a z$ as a single log

Omit Numerical Examples video 15:30 – 19:18.

Properties of Inverses of Logarithms: (result of the composition of logs and exponentials) (p. 426)

1. $a^{\log_a x} = x$

Ex. $5^{\log_5 7} =$

2. $\log_a(a^x) = x$

Ex. $\log_6(6^3) =$

Ex. $\log_r(r^{k+1}) =$

Calculator Hints:

1. When putting in a fraction as an exponent, you have to use ( ) around the fraction so that the calculator can "see" the entire fraction.
2. We'll learn how to put logs with bases other than 10 or *e* into the calculator in Section 4.4.

Assignments: Text: pp. 427 - 430 #1 – 12, 13 – 29 odd, 39 – 44, 57 – 69 odd, 79