

# 11-3 Contingency Tables

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## Definition

- ❖ **Contingency Table** (or two-way frequency table)  
a table in which frequencies correspond to two variables.  
(One variable is used to categorize rows, and a second variable is used to categorize columns.)

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## Definition

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**Contingency tables have at least two rows and at least two columns.**

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## Contingency Table

	Homicide	Robbery	Assault
Stranger	12	379	727
Acquaintance or Relative	39	106	642

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## Definition

### ❖ Test of Independence

tests the null hypothesis that there is no association between the row variable and the column variable.

(The null hypothesis is the statement that the row and column variables are independent.)

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## Assumptions

1. The sample data are randomly selected.
2. The null hypothesis  $H_0$  is the statement that the row and column variables are independent; the alternative hypothesis  $H_1$  is the statement that the row and variables are dependent.
3. For every cell in the contingency table, the expected frequency  $E$  is at least 5. (There is no requirement that every observed frequency must be at least 5.)

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## Tests of Independence

$H_0$ : The row variable is independent of the column variable

$H_1$ : The row variable is dependent (related to) the column variable

This procedure cannot be used to establish a direct cause-and-effect link between variables in question.

Dependence means only there is a relationship between the two variables.

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## Test of Independence Test Statistic

$$X^2 = \sum \frac{(O - E)^2}{E}$$

### Critical Values

1. Found in Table A-4 using

degrees of freedom =  $(r - 1)(c - 1)$

$r$  is the number of rows and  $c$  is the number of columns

2. Tests of Independence are always right-tailed.

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$$E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$



Total number of all observed frequencies  
in the table

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## Contingency Table

	Homicide	Robbery	Assault
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## Expected Frequency for Contingency Tables

$$E = \text{grand total} \cdot \frac{\text{row total}}{\text{grand total}} \cdot \frac{\text{column total}}{\text{grand total}}$$

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## Expected Frequency for Contingency Tables

$$E = \underbrace{\text{grand total}}_n \cdot \frac{\text{row total}}{\text{grand total}} \cdot \frac{\text{column total}}{\text{grand total}}$$

$\cdot$   $\underbrace{\hspace{10em}}_p$   
 (probability of a cell)

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### Expected Frequency for Contingency Tables

$$E = \frac{\text{grand total}}{n} \cdot \frac{\text{row total}}{\text{grand total}} \cdot \frac{\text{column total}}{\text{grand total}}$$

(probability of a cell)

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### Expected Frequency for Contingency Tables

$$E = \frac{\text{grand total}}{n} \cdot \frac{\text{row total}}{\text{grand total}} \cdot \frac{\text{column total}}{\text{grand total}}$$

(probability of a cell)

$$E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$

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Is the type of crime independent of whether the criminal is a stranger?

	Homicide	Robbery	Assault	
Stranger	12	379	727	
Acquaintance or Relative	39	106	642	

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Is the type of crime independent of whether the criminal is a stranger?

	Homicide	Robbery	Assault	
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$H_0$ : Type of crime is independent of knowing the criminal

$H_1$ : Type of crime is dependent with knowing the criminal

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Is the type of crime independent of whether the criminal is a stranger?

	Homicide	Robbery	Assault	Row Total
Stranger	12	379	727	1118
Acquaintance or Relative	39	106	642	787
Column Total	51	485	1369	

$H_0$ : Type of crime is independent of knowing the criminal

$H_1$ : Type of crime is dependent with knowing the criminal

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Is the type of crime independent of whether the criminal is a stranger?

	Homicide	Robbery	Assault	Row Total
Stranger	12	379	727	1118
Acquaintance or Relative	39	106	642	787
Column Total	51	485	1369	1905

$H_0$ : Type of crime is independent of knowing the criminal

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Column Total	51	485	1369	1905

$$E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$

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Is the type of crime independent of whether the criminal is a stranger?

	Homicide	Robbery	Assault	Row Total
Stranger	12 (29.93)	379	727	1118
Acquaintance or Relative	39	106	642	787
Column Total	51	485	1369	1905

$$E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$

$$E = \frac{(1118)(51)}{1905} = 29.93$$

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Is the type of crime independent of whether the criminal is a stranger?

	Homicide	Robbery	Assault	Row Total
Stranger	12 (29.93)	379 (284.64)	727 (803.43)	1118
Acquaintance or Relative	39 (21.07)	106 (200.36)	642 (565.57)	787
Column Total	51	485	1369	1905

$$E = \frac{(\text{row total})(\text{column total})}{(\text{grand total})}$$

$$E = \frac{(1118)(51)}{1905} = 29.93 \quad E = \frac{(1118)(485)}{1905} = 284.64$$

*etc.*

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Is the type of crime independent of whether the criminal is a stranger?

$$X^2 = \sum \frac{(O - E)^2}{E}$$

	Homicide	Robbery	Forgery
Stranger	12 (29.93) [10.741]	379 (284.64)	727 (803.43)
Acquaintance or Relative	39 (21.07)	106 (200.36)	642 (565.57)

$$\frac{(O - E)^2}{E}$$

Upper left cell:  $\frac{(O - E)^2}{E} = \frac{(12 - 29.93)^2}{29.93} = 10.741$

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Is the type of crime independent of whether the criminal is a stranger?

$$X^2 = \sum \frac{(O - E)^2}{E}$$

	Homicide	Robbery	Forgery
Stranger	12 (29.93) [10.741]	379 (284.64) [31.281]	727 (803.43) [7.271]
Acquaintance or Relative	39 (21.07) [15.258]	106 (200.36) [44.439]	642 (565.57) [10.329]

$$\frac{(O - E)^2}{E}$$

Upper left cell:  $\frac{(O - E)^2}{E} = \frac{(12 - 29.93)^2}{29.93} = 10.741$

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Acquaintance or Relative	39 (21.07) [15.258]	106 (200.36) [44.439]	642 (565.57) [10.329]

$$\frac{(O - E)^2}{E}$$

Test Statistic  $X^2 = 10.741 + 31.281 + \dots + 10.329 = 119.319$

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Test Statistic:  $\chi^2 = 119.319$

with  $\alpha = 0.05$  and  $(r - 1)(c - 1) = (2 - 1)(3 - 1) = 2$  degrees of freedom

Critical Value:  $\chi^2 = 5.991$  (from Table A-4)

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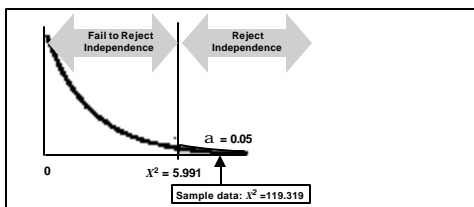
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Test Statistic:  $\chi^2 = 119.319$

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Critical Value:  $\chi^2 = 5.991$  (from Table A-4)



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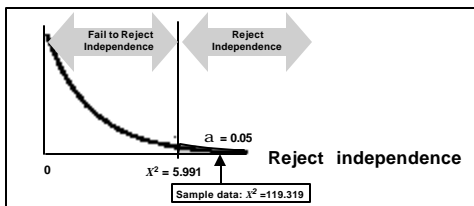
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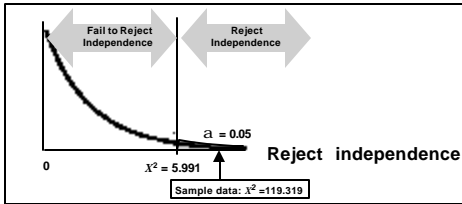
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Test Statistic:  $\chi^2 = 119.319$

with  $\alpha = 0.05$  and  $(r - 1)(c - 1) = (2 - 1)(3 - 1) = 2$  degrees of freedom

Critical Value:  $\chi^2 = 5.991$  (from Table A-4)



$H_0$  : The type of crime and knowing the criminal are independent  
 $H_1$  : The type of crime and knowing the criminal are dependent

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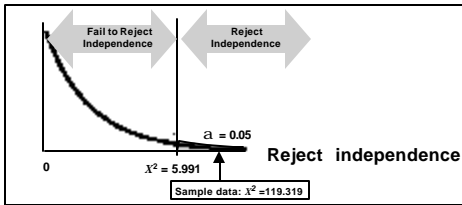
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Test Statistic:  $\chi^2 = 119.319$

with  $\alpha = 0.05$  and  $(r - 1)(c - 1) = (2 - 1)(3 - 1) = 2$  degrees of freedom

Critical Value:  $\chi^2 = 5.991$  (from Table A-4)



**It appears that the type of crime and knowing the criminal are related.**

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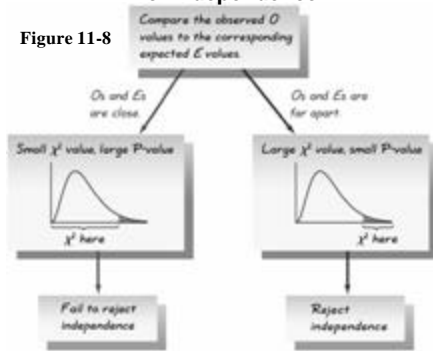
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### Relationships Among Components in $\chi^2$ Test of Independence

Figure 11-8



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# Definition

## Test of Homogeneity

tests the claim that *different populations* have the same proportions of some characteristics

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## Example - Test of Homogeneity

		New York	Chicago	Pittsburgh
Taxi has usable seat belt?	Yes	3	42	2
	No	74	87	70

Claim: The 3 cities have the same proportion of taxis with usable seat belts  
 $H_0$ : The 3 cities have the same proportion of taxis with usable seat belts  
 $H_a$ : The proportion of taxis with usable seat belts is not the same in all 3 cities

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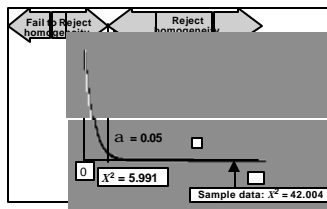
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## Example - Test of Homogeneity

		New York	Chicago	Pittsburgh
Taxi has usable seat belt?	Yes	3	42	2
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 $H_0$ : The 3 cities have the same proportion of taxis with usable seat belts  
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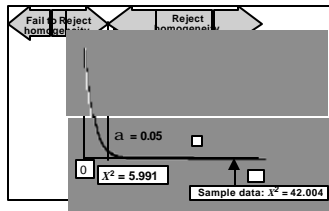
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### Example - Test of Homogeneity

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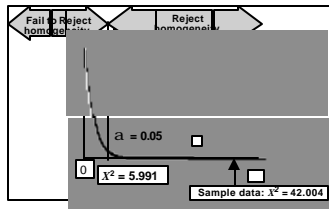
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### Example - Test of Homogeneity

		New York	Chicago	Pittsburgh
Taxi has usable seat belt?	Yes	3	42	2
	No	74	87	70

Claim: The 3 cities have the same proportion of taxis with usable seat belts  
 $H_0$ : The 3 cities have the same proportion of taxis with usable seat belts  
 $H_a$ : The proportion of taxis with usable seat belts is not the same in all 3 cities



There is sufficient evidence to warrant rejection of the claim that the 3 cities have the same proportion of usable seat belts in taxis; appears from Table Chicago has a much higher proportion.

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