

Measures of Variation

Section 3-3

Waiting Times of Bank Customers at Different Banks in minutes

Jefferson Valley Bank	6.5	6.6	6.7	6.8	7.1	7.3	7.4	7.7	7.7	7.7
Bank of Providence	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10.0

Waiting Times of Bank Customers at Different Banks in minutes

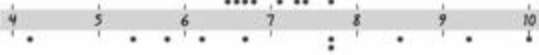
Jefferson Valley Bank	6.5	6.6	6.7	6.8	7.1	7.3	7.4	7.7	7.7	7.7
Bank of Providence	4.2	5.4	5.8	6.2	6.7	7.7	7.7	8.5	9.3	10.0

Jefferson Valley Bank Bank of Providence

Mean	7.15	7.15
Median	7.20	7.20
Mode	7.7	7.7
Midrange	7.10	7.10

Dotplots of Waiting Times

Jefferson Valley Bank (single waiting line)
Mean = 7.15
Median = 7.20
Mode = 7.7
Midrange = 7.10



Bank of Providence (multiple waiting lines)
Mean = 7.15
Median = 7.20
Mode = 7.7
Midrange = 7.10

Measures of Variation

Measures of Variation

Range

highest value – lowest value

Measures of Variation

Standard Deviation

- ❖ A measure of variation of the scores about the mean
- ❖ Average deviation from the mean
- ❖ Average 'distance' scores are from the mean

Rough Definition

Average "distance" the scores are from the mean

$$\frac{S(x - \bar{x})}{n}$$

2, 3, 3, 5, 7

2, 3, 3, 5, 7

$$\bar{x} = 4$$

2, 3, 3, 5, 7

$$\bar{x} = 4$$

$$(x - \bar{x}) = -2, -1, -1, 1, 3$$

2, 3, 3, 5, 7

$$\bar{x} = 4$$

$$(x - \bar{x}) = -2, -1, -1, 1, 3$$

$$\frac{S(x - \bar{x})}{n} = 0/5 = 0$$

Mean Absolute Deviation Formula

$$\frac{\sum |x - \bar{x}|}{n}$$

2, 3, 3, 5, 7

$$\bar{x} = 4$$

$$|x - \bar{x}| = 2, 1, 1, 1, 3$$

$$\frac{\sum |x - \bar{x}|}{n} = 8/5 = 1.6$$

Standard Deviation Formula for a Sample

Standard Deviation Formula for a Sample

$$S = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Formula 3-4

calculators can compute the
sample standard deviation of data

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Sample Standard Deviation Shortcut Formula

$$s = \sqrt{\frac{n(\sum x^2) - (\sum x)^2}{n(n-1)}}$$

Formula 3-5

calculators can compute the
sample standard deviation of data

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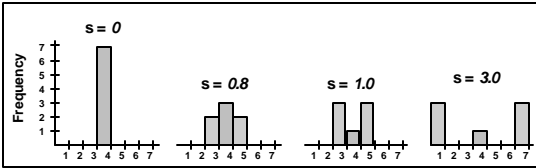
Important Properties of Standard Deviation

- ❖ A measure of variation of all values from the mean
- ❖ Usually positive; is zero (0) when all data are the same
- ❖ Value can increase dramatically with outliers
- ❖ Units are the same as the units of the original data

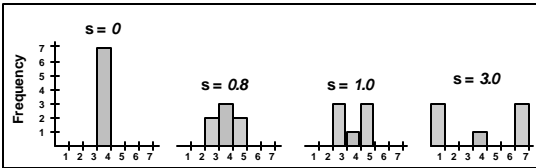
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**Same Means ($\bar{x} = 4$)
Different Standard Deviations**



**Same Means ($\bar{x} = 4$)
Different Standard Deviations**



When data is more varied, the standard deviation gets larger.

**Using Your Calculator to find
the standard deviation of a
data set**

6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7

Using Your Calculator to find the standard deviation of a data set

6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7

S = 0.48 minutes

Population Standard Deviation

$$S = \sqrt{\frac{\sum (x - \mu)^2}{N}}$$

calculators can compute the population standard deviation of data

Measures of Variation

Variance

Measures of Variation

Variance

standard deviation squared

Measures of Variation

Variance

standard deviation squared

Notation $\left\{ \begin{array}{l} S^2 \\ s^2 \end{array} \right.$ use square key
on calculator

Variance

$$s^2 = \frac{S(x - \bar{x})^2}{n - 1} \quad \text{Sample Variance}$$

$$S^2 = \frac{S(x - \mu)^2}{N} \quad \text{Population Variance}$$

Using Your Calculator to find the standard deviation of a data set

**6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7**

S = 0.48 minutes

Using Your Calculator to find the standard deviation of a data set

**6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7**

S = 0.48 minutes

S² = 0.23 minutes²

Notation

Sample

Textbook	→	s
Some graphics calculators	→	Sx
Some non-graphics calculators	→	xS_{n-1}

Notation

Sample

Population

Textbook → **s**

S ← Book

Some graphics calculators → **Sx**

Sx ← Some graphics calculators

Some non-graphics calculators → **xS_{n-1}**

xS_n ← Some non-graphics calculators

Notation

Sample

Population

Textbook → **s**

S ← Book

Some graphics calculators → **Sx**

Sx ← Some graphics calculators

Some non-graphics calculators → **xS_{n-1}**

xS_n ← Some non-graphics calculators

Articles in professional journals and reports often use SD for standard deviation and VAR for variance.

Round-off Rule for Measures of Variation

Carry one more decimal place than is present in the original set of data.

Round only the final answer, never in the middle of a calculation.

Standard Deviation from a Frequency Distribution

$$S = \sqrt{\frac{n [S(f \cdot x^2)] - [S(f \cdot x)]^2}{n(n-1)}}$$

Use the class midpoints as the x values.

Calculators can compute the standard deviation for a frequency distribution.

Standard Deviation and Variance of a Frequency Distribution

Quiz Scores	Frequency
0-4	2
5-9	5
10-14	8
15-19	11
20-24	7

Standard Deviation and Variance of a Frequency Distribution

Quiz Scores	Midpoints	Frequency
0-4	2	2
5-9	7	5
10-14	12	8
15-19	17	11
20-24	22	7

Standard Deviation and Variance of a Frequency Distribution

Quiz Scores	Midpoints	Frequency
0-4	2	2
5-9	7	5
10-14	12	8
15-19	17	11
20-24	22	7

$$S = 5.9$$

$$\sigma = 5.8$$

Standard Deviation and Variance of a Frequency Distribution

Quiz Scores	Midpoints	Frequency
0-4	2	2
5-9	7	5
10-14	12	8
15-19	17	11
20-24	22	7

$$S = 5.9$$

$$S^2 = 34.6$$

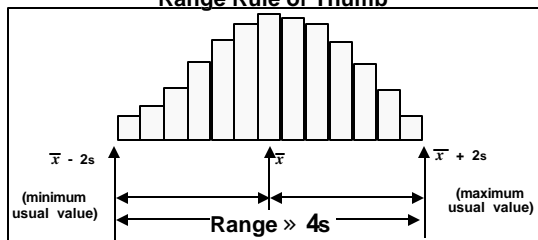
(using S not rounded)

$$\sigma = 5.8$$

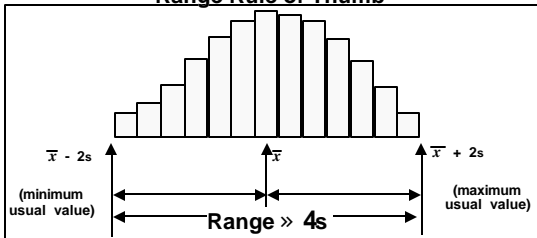
$$\sigma^2 = 33.5$$

(using S not rounded)

Estimation of Standard Deviation Range Rule of Thumb

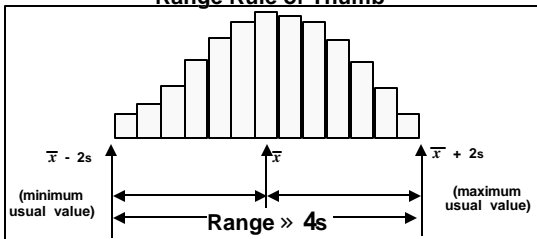


**Estimation of Standard Deviation
Range Rule of Thumb**



$$s \gg \frac{\text{Range}}{4}$$

**Estimation of Standard Deviation
Range Rule of Thumb**



$$s \gg \frac{\text{Range}}{4} = \frac{\text{highest value} - \text{lowest value}}{4}$$

Estimating the standard deviation using the Range Rule of Thumb

**6.5 6.6 6.7 6.8 7.1
7.3 7.4 7.7 7.7 7.7**

$$\begin{aligned}
 S &\sim \text{Range} / 4 = (7.7 - 6.5) / 4 \\
 &= 1.2 / 4 \\
 &= 0.3 \text{ min (estimate)} \\
 S &= 0.48 \text{ minutes (actual)}
 \end{aligned}$$

Rough Estimates of Usual Sample Values

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minimum 'usual' value » (mean) - 2 (standard deviation)

$$\text{minimum} \gg \bar{x} - 2(s)$$

Rough Estimates of Usual Sample Values

minimum 'usual' value » (mean) - 2 (standard deviation)

$$\text{minimum} \gg \bar{x} - 2(s)$$

maximum 'usual' value » (mean) + 2 (standard deviation)

$$\text{maximum} \gg \bar{x} + 2(s)$$

Rough Estimates of Usual Sample Values

Quiz Scores

$$\text{minimum} \gg \bar{x} - 2(s)$$

$$\text{minimum} \gg 14.4 - 2(5.9) = 2.6$$

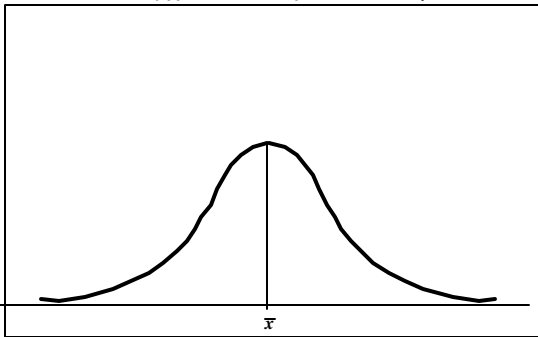
$$\text{maximum} \gg \bar{x} + 2(s)$$

$$\text{maximum} \gg 14.4 + 2(5.9) = 26.2$$

The Empirical Rule

FIGURE 3-4

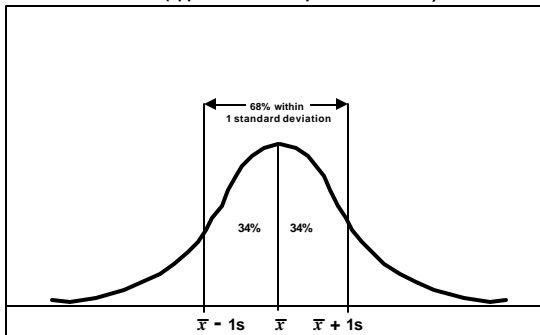
(applies to bell-shaped distributions)



The Empirical Rule

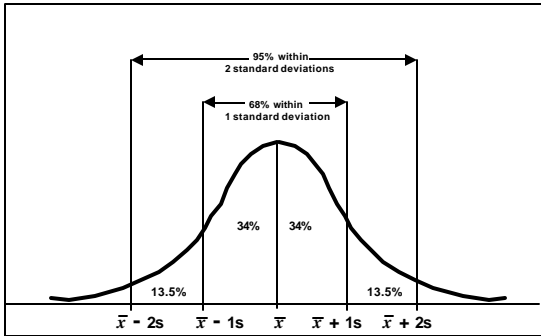
FIGURE 3-4

(applies to bell-shaped distributions)



The Empirical Rule (applies to bell-shaped distributions)

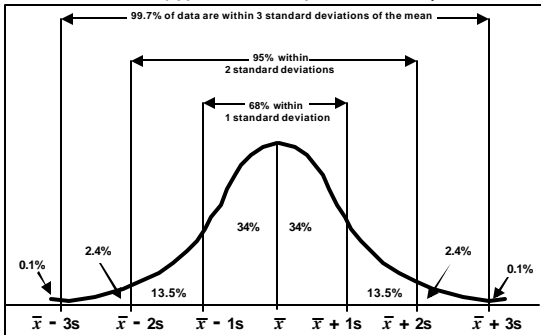
FIGURE 3-4



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The Empirical Rule (applies to bell-shaped distributions)

FIGURE 3-4



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Measures of Variation Summary

For typical data sets, it is unusual for a score to differ from the mean by more than 2 standard deviations.

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