

# Chapter 4

## Probability

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# Chapter 4

## Probability

- 4-1 Overview
- 4-2 Fundamentals
- 4-3 Addition Rule
- 4-4 Multiplication Rule: Basics

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# Overview

## Objectives

- ❖ develop sound understanding of probability values used in subsequent chapters
- ❖ develop basic skills necessary to solve probability values in a variety of circumstances

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# Overview

## Rare Event Rule for Inferential Statistics:

If, under a given assumption, the probability of a particular observed event is extremely small, we conclude that the assumption is probably not correct.

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# Overview

## Rare Event Rule for Inferential Statistics:

If, under a given assumption (the lottery is fair), the probability of a particular observed event is extremely small (winning lottery five times in a row), we conclude that the assumption is probably not correct.

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## 4-2 Fundamentals Definitions

- ❖ Event - any collection of results or outcomes from some procedure
- ❖ Simple event - any outcome or event that cannot be broken down into simpler components
- ❖ Sample space - all possible *simple* events

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## Notation

**P** - denotes a probability

**A, B, ...** - denote specific events

**P (A)** - denotes the probability of event A occurring

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## Basic Rules for Computing Probability

### Rule 1: Relative Frequency Approximation

Conduct (or observe) an experiment a large number of times, and count the number of times event A actually occurs, then an estimate of P(A) is

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## Basic Rules for Computing Probability

### Rule 1: Relative Frequency Approximation

Conduct (or observe) an experiment a large number of times, and count the number of times event A actually occurs, then an estimate of P(A) is

$$P(A) = \frac{\text{number of times A occurred}}{\text{number of times trial was repeated}}$$

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## Basic Rules for Computing Probability

### Rule 1: Relative Frequency Approximation

Conduct (or observe) an experiment a large number of times, and count the number of times event A actually occurs, then an estimate of  $P(A)$  is

$$P(A) \sim \frac{\text{number of times A occurred}}{\text{number of times trial was repeated}}$$

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## Basic Rules for Computing Probability

### Rule 2: Classical approach

(requires equally likely outcomes)

If a procedure has  $n$  different simple events, each with an equal chance of occurring, and event A can occur in  $s$  of these ways, then

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## Basic Rules for Computing Probability

### Rule 2: Classical approach

(requires equally likely outcomes)

If a procedure has  $n$  different simple events, each with an equal chance of occurring, and event A can occur in  $s$  of these ways, then

$$P(A) = \frac{s}{n} = \frac{\text{number of ways A can occur}}{\text{number of different simple events}}$$

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## Basic Rules for Computing Probability

### Rule 3: Subjective Probabilities

$P(A)$ , the probability of A, is found by simply guessing or estimating its value based on knowledge of the relevant circumstances.

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### Rule 1

The relative frequency approach is an approximation.

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### Rule 1

The relative frequency approach is an approximation.

### Rule 2

The classical approach is the actual probability.

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## Law of Large Numbers

As a procedure is repeated again and again, the relative frequency probability (from Rule 1) of an event tends to approach the actual probability.

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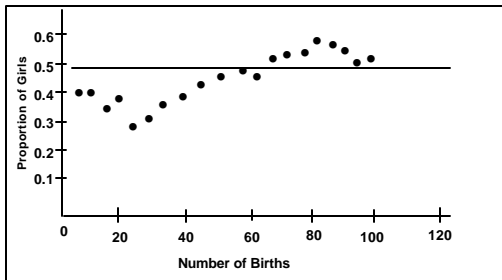
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## Illustration of Law of Large Numbers



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**Example:** Find the probability that a randomly selected person will be struck by lightning this year.

The sample space consists of two simple events: the person is struck by lightning or is not. Because these simple events are not equally likely, we can use the relative frequency approximation (Rule 1) or subjectively estimate the probability (Rule 3). Using Rule 1, we can research past events to determine that in a recent year 377 people were struck by lightning in the US, which has a population of about 274,037,295. Therefore,

$$P(\text{struck by lightning in a year}) \gg \\ 377 / 274,037,295 \gg 1/727,000$$

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**Example:** On an ACT or SAT test, a typical multiple-choice question has 5 possible answers. If you make a random guess on one such question, what is the probability that your response is wrong?

There are 5 possible outcomes or answers, and there are 4 ways to answer incorrectly. Random guessing implies that the outcomes in the sample space are equally likely, so we apply the classical approach (Rule 2) to get:

$$P(\text{wrong answer}) = 4 / 5 = 0.8$$

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## Probability Limits

- ❖ The probability of an impossible event is 0.
- ❖ The probability of an event that is certain to occur is 1.

$$0 \leq P(A) \leq 1$$

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## Probability Limits

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$$0 \leq P(A) \leq 1$$

↓  
Impossible to occur

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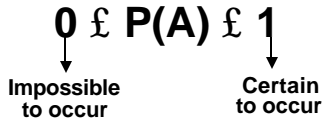
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# Probability Limits

- ❖ The probability of an impossible event is 0.
- ❖ The probability of an event that is certain to occur is 1.



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# Possible Values for Probabilities

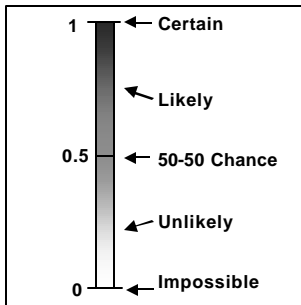


Figure 4-2

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# Possible Values for Probabilities

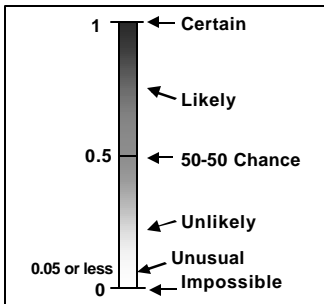


Figure 4-2

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# Complementary Events

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# Complementary Events

The complement of event A, denoted by  $\bar{A}$ , consists of all outcomes in which event A does NOT occur.

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# Complementary Events

The complement of event A, denoted by  $\bar{A}$ , consists of all outcomes in which event A does NOT occur.

$$P(A) \qquad P(\bar{A})$$

(read "not A")

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### Example: Testing Corvettes

The General Motors Corporation wants to conduct a test of a new model of Corvette. A pool of 50 drivers has been recruited, 20 of whom are men. When the first person is selected from this pool, what is the probability of *not* getting a male driver?

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### Example: Testing Corvettes

The General Motors Corporation wants to conduct a test of a new model of Corvette. A pool of 50 drivers has been recruited, 20 of whom are men. When the first person is selected from this pool, what is the probability of *not* getting a male driver?

Because 20 of the 50 subjects are men, it follows that 30 of the 50 subjects are women so,

$$\begin{aligned} P(\text{not selecting a man}) &= P(\overline{\text{man}}) \\ &= P(\text{woman}) \\ &= \frac{30}{50} = 0.6 \end{aligned}$$

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## Rounding Off Probabilities

❖ give the exact fraction or decimal  
or

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## Rounding Off Probabilities

- ❖ give the exact fraction or decimal  
or
- ❖ round off the final result to  
three significant digits

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## Rounding Off Probabilities

Examples:

- ❖ give the exact fraction or  
decimal

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## Rounding Off Probabilities

Examples:

- ❖ give the exact fraction or  
decimal

$P(\text{drawing a red ball}) = 1/3$

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## Rounding Off Probabilities

Examples:

❖ give the exact fraction or decimal

$$P(\text{drawing a red ball}) = 1/3$$

$$P(\text{exactly 2 boys in a 3-child family}) = 3/8 = 0.375$$

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## Rounding Off Probabilities

Examples:

❖ round the final result to three significant digits

$$P(\text{drawing a red ball}) = 1/3 = 0.333$$

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## Rounding Off Probabilities

Examples:

❖ round the final result to three significant digits

$$P(\text{drawing a red ball}) = 1/3 = 0.333$$

$$P(\text{struck by lightning last year}) = 0.00000143$$

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### 3 Significant Digits

- a.) 0.000123 (six decimal places)
- b.) 0.123 (three decimal places)
- c.) 0.102 (three decimal places)
- d.) 0.00102 (five decimal places)

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### 3 Significant Digits

- a) 0.000123 (six decimal places)
- b) 0.123 (three decimal places)
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- d) 0.00102 (five decimal places)

Why not use just 3 digits?

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### 3 Significant Digits

- a) 0.000123 (six decimal places)
- b) 0.123 (three decimal places)
- c) 0.102 (three decimal places)
- d) 0.00102 (five decimal places)

Why not use just 3 digits?

$$0.000123 = 0.000$$

which would suggest the event is impossible

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# Only Exception

**$0.99999999 = 0.999+$**

**since**

**$0.99999999 = 1.000$**

**Suggests the event is CERTAIN to occur**

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