4 - 4

Multiplication Rule: Basics

Finding the Probability of Two or More Selections

- Multiple selections
- Multiplication Rule

Notation

\[ P(A \text{ and } B) = P(\text{event A occurs in a first trial and event B occurs in a second trial}) \]
Tree Diagram of Test Answers

\[
\begin{align*}
T & \quad a \quad Ta \\
& \quad b \quad Tb \\
& \quad c \quad Tc \\
& \quad d \quadTd \\
& \quad e \quad Te \\
F & \quad a \quad Fa \\
& \quad b \quad Fb \\
& \quad c \quad Fc \\
& \quad d \quad Fd \\
& \quad e \quad Fe \\
\end{align*}
\]

\[P(T) = \frac{1}{2}\]

Tree Diagram of Test Answers

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& \quad e \quad Fe \\
\end{align*}
\]

\[P(T) = \frac{1}{2}, \quad P(c) = \frac{1}{5}\]
Tree Diagram of Test Answers

\[ P(T) = \frac{1}{2} \quad P(c) = \frac{1}{5} \quad P(T \text{ and } c) = \frac{1}{10} \]

\[
\begin{array}{c}
\text{T} \\
\text{a} \\
\text{b} \\
\text{c} \\
\text{d} \\
\text{e} \\
\text{T_a} \\
\text{T_b} \\
\text{T_c} \\
\text{T_d} \\
\text{T_e} \\
\text{F} \\
\text{F_a} \\
\text{F_b} \\
\text{F_c} \\
\text{F_d} \\
\text{F_e} \\
\end{array}
\]

P (both correct) = P (T \text{ and } c)
\[ P(\text{both correct}) = P(T \text{ and } c) = \frac{1}{10} \cdot \frac{1}{2} \cdot \frac{1}{5} \]

**Multiplication Rule**

**INDEPENDENT EVENTS**
Notation for Conditional Probability

\[ P(B|A) \] represents the probability of event \( B \) occurring after it is assumed that event \( A \) has already occurred (read \( B \mid A \) as “\( B \) given \( A \)”).

Definitions

- **Independent Events**
  Two events \( A \) and \( B \) are independent if the occurrence of one does not affect the probability of the occurrence of the other.

- **Dependent Events**
  If \( A \) and \( B \) are not independent, they are said to be dependent.

Formal Multiplication Rule

- \( P(A \text{ and } B) = P(A) \cdot P(B \mid A) \)
- If \( A \) and \( B \) are independent events, \( P(B \mid A) \) is really the same as \( P(B) \)
Applying the Multiplication Rule

P(A and B) = P(A) \cdot P(B \mid A)

Independent Events

• Two selections
• With replacement

P (both good) = \frac{4}{5} \cdot \frac{4}{5} = \frac{16}{25} = 0.64
• Two selections
• Without replacement

\[ P \text{ (both good)} = \]

\[ P \text{ (good)} \cdot P\text{(good)} = \]
Two selections

Without replacement

\[ P \text{ (both good)} = \]

\[ P \text{ (good)} \cdot P \text{ (good)} = \]

\[ \frac{4}{5} \]
Two selections
Without replacement

\[ P(\text{both good}) = \frac{4}{5} \cdot \frac{3}{4} = \frac{12}{20} = 0.60 \]

(assume good was selected)

DEPENDENT EVENTS

Two selections
Without replacement

\[ P(\text{both good}) = \frac{4}{5} \cdot \frac{3}{4} = \frac{12}{20} = 0.60 \]

(assume good was selected)

Intuitive Multiplication

When finding the probability that event A occurs in one trial and B occurs in the next trial, multiply the probability of event A by the probability of event B, but be sure that the probability of event B takes into account the previous occurrence of event A.
Find the probability of drawing two cards from a shuffled deck of cards such that the first is an Ace and the second is a King. (The cards are drawn without replacement.)

- $P(\text{Ace on first card}) = \frac{4}{52}$
- $P(\text{King | Ace}) = \frac{4}{51}$
- $P(\text{drawing Ace, then a King}) = \frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652} = 0.00603$
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**Example:** On a TV program it was reported that there is a 60% success rate for those who try to stop smoking through hypnosis. Find the probability that for 8 randomly selected smokers who undergo hypnosis, they all successfully quit smoking.

$P(\text{all 8 quit smoking}) = \text{success rate}^8 = 0.6^8$
Example: On a TV program it was reported that there is a 60% success rate for those who try to stop smoking through hypnosis. Find the probability that for 8 randomly selected smokers who undergo hypnosis, they all successfully quit smoking.

\[
P(\text{all 8 quit smoking}) = P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) = (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60)
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$$P(\text{all 8 quit smoking}) = P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) \cdot P(\text{quit}) = (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60) \cdot (0.60)$$

or

$$0.60^8 = 0.0168$$

Small Samples from Large Populations

If a sample size is no more than 5% of the size of the population, treat the selections as being independent (even if the selections are made without replacement, so they are technically dependent).

Example: If Houston has an annual car-theft rate of 4.5%, find the probability that among 4 randomly selected cars, all are stolen during a given year. (There are 970,000 cars in Houston.)
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P(\text{all 4 cars stolen}) = P(\text{stolen}) \times P(\text{stolen}) \times P(\text{stolen}) \times P(\text{stolen})
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\[
\text{INDEPENDENT} \\
\frac{43650}{970000} = 0.00004100625
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\[
\text{DEPENDENT} \\
(0.045)^4 = \frac{43650 \times 43649 \times 43648 \times 43647}{970000 \times 969999 \times 969998 \times 969997} = 0.00000410086725
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\[ P(\text{all 4 cars stolen}) = P(\text{stolen}) \times P(\text{stolen}) \times P(\text{stolen}) \times P(\text{stolen}) = (0.045)^4 = \frac{43650}{970000} \times \frac{43649}{969999} \times \frac{43648}{969998} \times \frac{43647}{969997} = 0.000004100625 \]

Independence

You must treat a problem as independent when:

- you do not have the sample or population size, and
- you have only a percentage (probability) of the individual characteristic