Chapter 5
Probability Distributions

5-1 Overview

5-2 Random Variables

5-3 Binomial Probability Distributions

5-4 Mean, Variance, Standard Deviation for the Binomial Distribution

Overview

This chapter will deal with the construction of

probability distributions

by combining the methods of Chapter 3 with the those of Chapter 4.

Probability Distributions will describe what will *probably* happen instead of what actually *did* happen.

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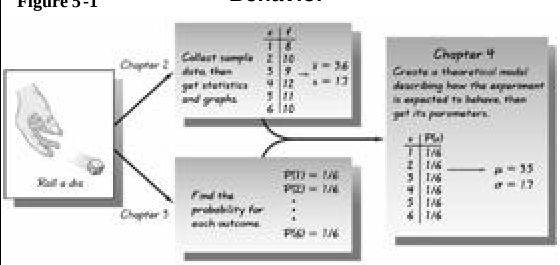
Combining Descriptive Statistics Methods and Probabilities to Form a Theoretical Model of Behavior

Figure 5-1



5-2

Random Variables

Definitions

❖ **Random Variable**

a variable (typically represented by x) that has a single numerical value, determined by chance, for each outcome of a procedure

❖ **Probability Distribution**

a graph, table, or formula that gives the probability for each value of the random variable

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Probability Distribution
Number of Girls Among Fourteen Newborn Babies

x	$P(x)$
0	0.000
1	0.001
2	0.006
3	0.022
4	0.061
5	0.122
6	0.183
7	0.209
8	0.183
9	0.122
10	0.061
11	0.022
12	0.006
13	0.001
14	0.000

Definitions

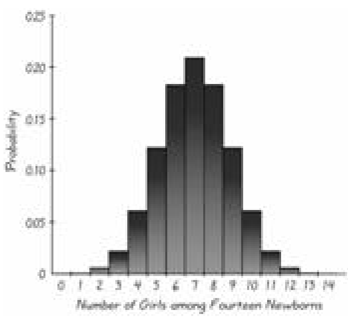
- **<u>Discrete</u> random variable**

  has either a finite number of values or countable number of values, where 'countable' refers to the fact that there might be infinitely many values, but they result from a counting process.

- **<u>Continuous</u> random variable**

  has infinitely many values, and those values can be associated with measurements on a continuous scale with no gaps or interruptions.

Probability Histogram



Requirements for Probability Distribution

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$$\sum P(x) = 1$$

where x assumes all possible values

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Requirements for Probability Distribution

$$\sum P(x) = 1$$

where x assumes all possible values

$$0 \text{ £ } P(x) \text{ £ } 1$$

for every value of x

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Mean, Variance and Standard Deviation of a Probability Distribution

Formula 5-1

$$\mu = \text{S } [x \cdot P(x)]$$

Formula 5-2

$$\text{s}^2 = \text{S } [(x - \mu)^2 \cdot P(x)]$$

Formula 5-3

$$\text{s}^2 = [\text{S } x^2 \cdot P(x)] - \mu^2 \quad \text{(shortcut)}$$

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Mean, Variance and Standard Deviation of a Probability Distribution

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Formula 5-2

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Formula 5-3

$$s^2 = [S\ x^2 \cdot P(x)] - \mu^2 \quad \text{(shortcut)}$$

Formula 5-4

$$s = \sqrt{[S\ x^2 \cdot P(x)] - \mu^2}$$

Mean, Variance and Standard Deviation of a Probability Distribution

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Formula 5-4

$$s = \sqrt{[S\ x^2 \cdot P(x)] - \mu^2}$$

Triola, Essentials of Statistics, Third Edition. Copyright 2008. Pearson Education, Inc.

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Roundoff Rule for μ, s^2, and s

Round results by carrying one more decimal place than the number of decimal places used for the random variable *x*. If the values of *x* are integers, round μ, s^2, and s to one decimal place.

Usual Sample Values

minimum = m - 2(s)

maximum = m + 2(s)

Probability Distribution
Number of Girls Among Fourteen Newborn Babies

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10	0.061
11	0.022
12	0.006
13	0.001
14	0.000

Usual Sample Values

minimum = 7.0 - 2(1.9) = 3.2

maximum = 7.0 + 2(1.9) = 10.8

Using the Rare Event Rule

If, under a given assumption (such as the assumption that boys and girls are equally likely), the probability of a particular observed event (such as 13 girls in 14 births) is extremely small, we conclude that the assumption was probably not correct (boys and girls NOT equally likely; the gender selection technique did have an effect).

Using Probabilities to Determine When Results Are Unusual

X **is unusually high if with** ***x*** **successes among** ***n*** **trials, P(x or more) is very small (such as 0.05 or less)**

X **is unusually low if with** ***x*** **successes among** ***n*** **trials, P(x or fewer) is very small (such as 0.05 or less)**

Probability Distribution
Number of Girls Among Fourteen Newborn Babies

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1	0.001
2	0.006
3	0.022
4	0.061
5	0.122
6	0.183
7	0.209
8	0.183
9	0.122
10	0.061
11	0.022
12	0.006
13	0.001
14	0.000

Definition

Expected Value

The average value of outcomes

$$E = \Sigma [x \cdot P(x)]$$

$$E = \Sigma [x \cdot P(x)]$$

Event
Win
Lose

$$E = \Sigma [x \cdot P(x)]$$

Event	x
Win	$499
Lose	- $1

$$E = \Sigma\ [x \cdot P(x)]$$

Event	x	P(x)
Win	$499	0.001
Lose	- $1	0.999

$$E = \Sigma\ [x \cdot P(x)]$$

Event	x	P(x)	x • P(x)
Win	$499	0.001	0.499
Lose	- $1	0.999	- 0.999

$$E = \Sigma\ [x \cdot P(x)]$$

Event	x	P(x)	x • P(x)
Win	$499	0.001	0.499
Lose	- $1	0.999	- 0.999

E = -$.50

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