5-3

Binomial Probability Distributions

Definition

Binomial Probability Distribution

1. The procedure must have a *fixed number of trials.*
2. The trials must be *independent.* (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
3. Each trial must have all outcomes classified into *two categories*.
4. The probabilities must remain *constant* for each trial.

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Notation for Binomial Probability Distributions

n = fixed number of trials

x = specific number of successes in n trials

p = probability of *success* in *one* of n trials

q = probability of *failure* in *one* of n trials ($q = 1 - p$)

$P(x)$ = probability of getting exactly x success among n trials

Be sure that x and p both refer to the same category being called a success.

Probability Distribution
Number of Girls Among Fourteen Newborn Babies

x	$P(x)$
0	0.000
1	0.001
2	0.006
3	0.022
4	0.061
5	0.122
6	0.183
7	0.209
8	0.183
9	0.122
10	0.061
11	0.022
12	0.006
13	0.001
14	0.000

Method 1

Binomial Probability Formula

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- $P(x) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x}$

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- $P(x) = \frac{n!}{(n-x)!\,x!} \cdot p^x \cdot q^{n-x}$
- $P(x) = {}_nC_x \cdot p^x \cdot q^{n-x}$

for calculators with ${}_nC_r$ key, where $r = x$

Example: Find the probability of getting exactly 3 correct responses among 5 different requests from AT&T directory assistance. Assume in general, AT&T is correct 90% of the time.

This is a binomial experiment where:

n = 5
x = 3
p = 0.90
q = 0.10

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Example: Find the probability of getting exactly 3 correct responses among 5 different requests from AT&T directory assistance. Assume in general, AT&T is correct 90% of the time.

This is a binomial experiment where:

n = 5
x = 3
p = 0.90
q = 0.10

Using the binomial probability formula to solve:

$P(3) = {}_5C_3 \cdot 0.9^3 \cdot 0.1^2 = 0.0729$

Method 2

Table A-1 in Appendix A

For n = 15 and p = 0.10

Table A-1

n	x	P(x)
15	0	0.206
	1	0.343
	2	0.267
	3	0.129
	4	0.043
	5	0.010
	6	0.002
	7	0.0+
	8	0.0+
	9	0.0+
	10	0.0+
	11	0.0+
	12	0.0+
	13	0.0+
	14	0.0+
	15	0.0+

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For n = 15 and p = 0.10

Table A-1

n	x	P(x)
15	0	0.206
	1	0.343
	2	0.267
	3	0.129
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	8	0.0+
	9	0.0+
	10	0.0+
	11	0.0+
	12	0.0+
	13	0.0+
	14	0.0+
	15	0.0+

For $n = 15$ and $p = 0.10$

Table A-1 **Binomial Probability Distribution**

n	x	$P(x)$
15	0	0.206
	1	0.343
	2	0.267
	3	0.129
	4	0.043
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	13	0.0+
	14	0.0+
	15	0.0+

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0	0.206
1	0.343
2	0.267
3	0.129
4	0.043
5	0.010
6	0.002
7	0.000
8	0.000
9	0.000
10	0.000
11	0.000
12	0.000
13	0.000
14	0.000
15	0.000

Example: **Using Table A-1 for n = 5 and p = 0.90, find the following:**

a) The probability of exactly 3 successes

b) The probability of at least 3 successes

a) P(3) = 0.073

b) P(at least 3) = P(3 or 4 or 5)

= P(3) or P(4) or P(5)

= 0.073 + 0.328 + 0.590

= 0.991

Method 3

Using Technology

- **STATDISK**
- **Minitab**
- **Excel**
- **TI-83/84**

Binomial Probability Formula

$$P(x) = \underbrace{\frac{n!}{(n-x)!\,x!}}_{\text{Number of outcomes with exactly } x \text{ successes among } n \text{ trials}} \cdot p^x \cdot q^{n-x}$$

Binomial Probability Formula

$$P(x) = \underbrace{{}_nC_r}_{\text{Number of outcomes with exactly } x \text{ successes among } n \text{ trials}} \cdot p^x \cdot q^{n-x}$$

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Binomial Probability Formula

$$P(x) = \underbrace{{}_nC_r}_{\text{Number of outcomes with exactly } x \text{ successes among } n \text{ trials}} \cdot \underbrace{p^x \cdot q^{n-x}}_{\text{Probability of } x \text{ successes among } n \text{ trials for any one particular order}}$$

Example

Nine percent of men and 0.25% of women cannot distinguish between red and green. This is the type of color blindness that causes problems with traffic signals. If six men are randomly selected for a study of traffic signal perceptions, find the probability that exactly two of them cannot distinguish between red and green.