Review for Exam #2

Chapters 4 and 5

Fundamentals of Probability

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Rule 1

The relative frequency approach gives an approximation of a probability.

Rule 2

The classical approach is the actual probability.

Rule 1
Relative frequency approach
Throwing a die 100 times and getting 15 threes

P(3) = 0.150

Rule 2
Classical approach
P(3 on a die) = 1/6 = 0.167

Probability Limits

- **The probability of an impossible event is 0.**
- **The probability of an event that is certain to occur is 1.**

0 £ P(A) £ 1

Impossible to occur ↓ (0) — **Certain to occur** ↓ (1)

- **A probability value must be a number between 0 and 1.**

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Complementary Events

The complement of event A, denoted by $\bar{A}$, consists of all outcomes in which event A does not occur.

P(A)

P($\bar{A}$)
(read “not A”)

Rounding Off Probabilities

Examples:

- **give the exact fraction or decimal**

or

- **round the final result to three significant digits**

P(struck by lightning last year) » 0.00000143

Definitions

Compound Event

Any event combining 2 or more events

Notation

P(A or B) = P (event A occurs <u>or</u> event B occurs <u>or</u> they both occur)

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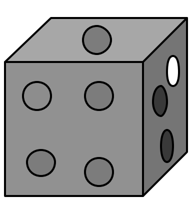
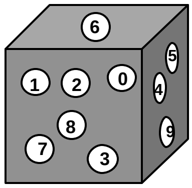
Disjoint Events



A = Green ball
B = Blue ball } **disjoint events**

$$P(A \text{ or } B) = P(A) + P(B) = \frac{4}{8} + \frac{1}{8} = \frac{5}{8}$$

Not Disjoint Events



A = Even number
B = Number greater than 5
} Overlapping events; some counted twice

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) = \frac{5}{10} + \frac{4}{10} - \frac{2}{10} = \frac{7}{10}$$

(0)(2)(4)(6)(8) (6)(7)(8)(9) (6)&(8) counted twice

Contingency Table

	Homicide	Robbery	Assault	Totals
Stranger	12	379	727	1118
Acqu. or Rel.	39	106	642	787
Unknown	18	20	57	95
Totals	69	505	1426	2000

Find the probability of randomly selecting one person from this group and getting someone who was robbed or was assaulted.

$$P(\text{robbed or assaulted}) = \frac{505}{2000} + \frac{1426}{2000} = \frac{1931}{2000} = 0.966$$

* Disjoint Events *

Contingency Table

	Homicide	Robbery	Assault	Totals
Stranger	12	379	727	1118
Acqu. or Rel.	39	106	642	787
Unknown	18	20	57	95
Totals	69	505	1426	2000

Find the probability of randomly selecting one person from this group and getting someone who was robbed or was a stranger.

$$P(\text{robbed or a stranger}) = \frac{505}{2000} + \frac{1118}{2000} - \frac{379}{2000} = \frac{1244}{2000} = 0.622$$

* * NOT Disjoint Events **

Complementary Events

$P(A)$ and $P(\bar{A})$ are disjoint events

All simple events are either in A or $\bar{A}$.

$$P(A) + P(\bar{A}) = 1$$

Finding the Probability of Two or More Selections

- ❖ Multiple selections
- ❖ Multiplication Rule

Definitions

Independent Events

Two events A and B are independent if the occurrence of one does not affect the probability of the occurrence of the other.

Dependent Events

If A and B are not independent, they are said to be dependent.

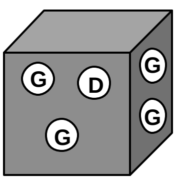
Find the probability of drawing two cards from a shuffled deck of cards such that the first is an Ace and the second is a King. (The cards are drawn <u>without replacement</u>.)

- P(Ace on first card) = $\frac{4}{52}$
- P(King | Ace) = $\frac{4}{51}$
- P(drawing Ace, then a King) = $\frac{4}{52} \cdot \frac{4}{51} = \frac{16}{2652}$

$= 0.00603$

DEPENDENT EVENTS

Independent Events



- Two selections
- <u>With</u> replacement

P (both good) =

P (good <u>and</u> good) =

$\frac{4}{5} \cdot \frac{4}{5} = \frac{16}{25} = 0.64$

Example: On a TV program it was reported that there is a 60% success rate for those who try to stop smoking through hypnosis. Find the probability that for 8 randomly selected smokers who undergo hypnosis, they all successfully quit smoking.

P(all 8 quit smoking) =

P(quit) P(quit) P(quit) P(quit) P(quit) P(quit) P(quit) P(quit) =

(0.60) • (0.60) • (0.60) • (0.60) • (0.60) • (0.60) • (0.60) • (0.60)

or

$0.60^8 = 0.0168$

Small Samples from Large Populations

If small sample is drawn from large population (if n £ 5% of N), you can treat the events as independent.

Independence

You must treat a problem as independent when:

- you do not have the sample or population size, and
- you have only a percentage (probability) of the individual characteristic

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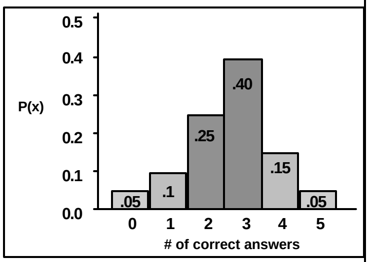
Probability Distribution

Probability Distribution

x (# of correct)	P(x)
0	.05
1	.10
2	.25
3	.40
4	.15
5	.05



Probability Histogram

Requirements for Probability Distribution

$$\Sigma P(x) = 1$$

where x assumes all possible values

$$0 \le P(x) \le 1$$

for every value of x

Mean, Variance and Standard Deviation of a Probability Distribution

Mean

$$\mu = \Sigma\, x \cdot P(x)$$

Variance

$$\sigma^2 = \Sigma\, [(x - \mu)^2 \cdot P(x)]$$

Variance

$$\sigma^2 = [\Sigma\, x^2 \cdot P(x)] - \mu^2 \text{ (shortcut)}$$

Standard Deviation

$$\sigma = \sqrt{[\Sigma\, x^2 \cdot P(x)] - \mu^2}$$

Mean, Standard Deviation and Variance of Probability Distribution

x	P(x)
0	.05
1	.10
2	.25
3	.40
4	.15
5	.05

$\mu = 2.7$

$S = 1.2$

$S^2 = 1.3$

$$E = S\ [x \cdot P(x)]$$

Event	x	P(x)	x • P(x)
Win	$499	0.001	0.499
Lose	- $1	0.999	-0.999

E = -$.50

Binomial Experiment

Definition

1. The procedure must have a ***fixed number of trials***.
2. The trials must be ***independent***. (The outcome of any individual trial doesn't affect the probabilities in the other trials.)
3. Each trial must have all outcomes classified into ***two categories***.
4. The probabilities must remain ***constant*** for each trial.

For n = 15 and p = 0.10

Table A-1

n	x	P(x)
15	0	0.206
	1	0.343
	2	0.267
	3	0.129
	4	0.043
	5	0.010
	6	0.002
	7	0.0+
	8	0.0+
	9	0.0+
	10	0.0+
	11	0.0+
	12	0.0+
	13	0.0+
	14	0.0+
	15	0.0+

For n = 15 and p = 0.10

Table A-1

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	14	0.0+
	15	0.0+

For n = 15 and p = 0.10

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	14	0.0+
	15	0.0+

Binomial Probability Distribution

x	P(x)
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12	0.000
13	0.000
14	0.000
15	0.000

Example: US Air has 20% of all domestic flights and one year had 4 of 7 consecutive major air crashes in the United States. Assuming that airline crashes are independent and random events, find the probability that when seven airliners crash, at least four of them are from US Air.

According to the definition, this is a binomial experiment.

$n = 7$

$p = 0.20$

$q = 0.80$

$x = 4, 5, 6, 7$

Table A-1 can be used.

$P(4,5,6,7) = 0.029 + 0.004 + 0^{+} + 0^{+} = 0.033$

Binomial Probability Formula

$$P(x) = \frac{n!}{(n-x)!\,x!} \cdot p^{x} \cdot q^{n-x}$$

Binomial Probability Formula

$$P(x) = {}_{n}C_{r} \cdot p^{x} \cdot q^{n-x}$$

${}_{n}C_{r}$: Number of outcomes with exactly *x* successes among *n* trials

$p^{x} \cdot q^{n-x}$: Probability of *x* successes among *n* trials for any one particular order

Example: Find the probability of getting exactly 3 left-handed students in a class of 20 if 10% of us are left-handed.

This is a binomial experiment where:

n = 20

x = 3

p = .10

q = .90

Table A-1 <u>cannot</u> be used; therefore, we must use the binomial formula.

$$P(3) = \frac{20!}{17!\,3!} \cdot 0.1^{3} \cdot 0.9^{17} = 0.190$$

Example: Find the probability of getting exactly 3 left-handed students in a class of 20 if 10% of us are left-handed.

This is a binomial experiment where:

n = 20

x = 3

p = .10

q = .90

Table A-1 <u>cannot</u> be used; therefore, we must use the binomial formula.

$$P(3) = {}_{20}C_{3} \cdot 0.1^{3} \cdot 0.9^{17} = 0.190$$

For a <u>Binomial</u> Distribution:

Mean $\mu = n \cdot p$

Variance $\sigma^{2} = n \cdot p \cdot q$

Standard Deviation $\sigma = \sqrt{n \cdot p \cdot q}$

Example: **US Air has 20% of all domestic flights. What is considered the 'unusual' number of US Air crashes out of seven randomly selected crashes?**

We previously found for this binomial distribution,

μ = 1.4 crashes

s = 1.1 crashes

μ - 2 s = 1.4 - 2(1.1) = - 0.8 (or 0)

μ + 2 s = 1.4 + 2(1.1) = 3.6

The usual number of US Air crashes out of seven randomly selected crashes should be between -0.8 (or 0) and 3.6.

Four crashes would be unusual!