

Chapter 6

Normal Probability Distributions

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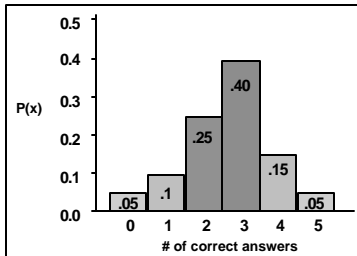
- 6-1 Overview
- 6-2 The Standard Normal Distribution
- 6-3 Applications of Normal Distributions
- 6-2 and 6-3: Normal Distributions: Finding Values
- 6-5 The Central Limit Theorem

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Review

x (# of correct)	$P(x)$
0	.05
1	.10
2	.25
3	.40
4	.15
5	.05

Probability Distribution



Probability Histogram

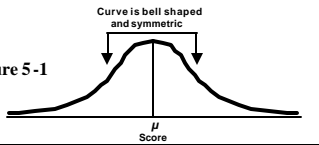
Overview

- ❖ Continuous random variable
- ❖ Normal distribution

Overview

- ❖ Continuous random variable
- ❖ Normal distribution

Figure 5-1

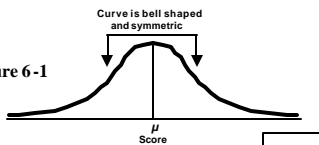


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Overview

- ❖ Continuous random variable
- ❖ Normal distribution

Figure 6-1



Formula 6-1

$$y = \frac{e^{-\frac{1}{2}\left(\frac{x-\mu}{S}\right)^2}}{S\sqrt{2\pi}}$$

6-2

The Standard Normal Distribution

Definitions

- ❖ **Density Curve**
(or probability density function)
the graph of a continuous probability distribution

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Definitions

- ❖ **Density Curve**
(or probability density function)
the graph of a continuous probability distribution
1. The total area under the curve must equal 1.
 2. Every point on the curve must have a vertical height that is 0 or greater.

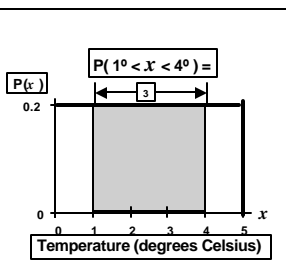
Because the total area under the density curve is equal to 1, there is a correspondence between area and probability.

Definition

- ❖ **Uniform Distribution**
a probability distribution in which the continuous random variable values are spread evenly over the range of possibilities; the graph results in a rectangular shape.

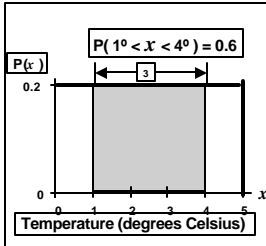
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Uniform Distributions

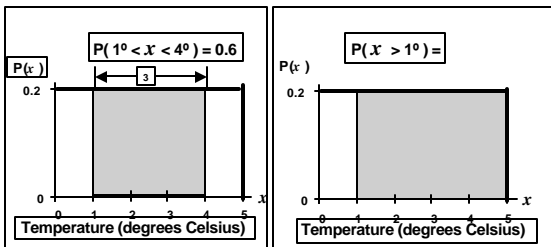


Uniform Distributions

Shaded Area = $L \cdot W = 3 \cdot 0.2 = 0.6$



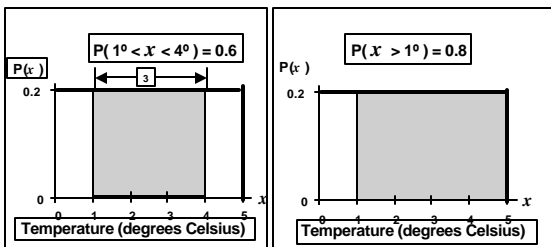
Uniform Distributions



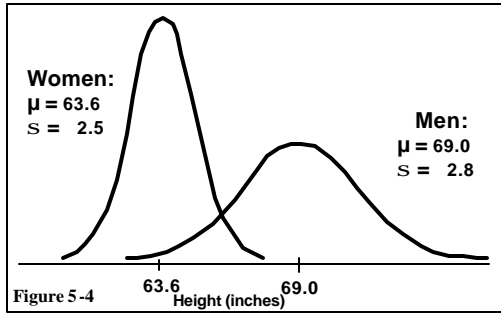
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Uniform Distributions

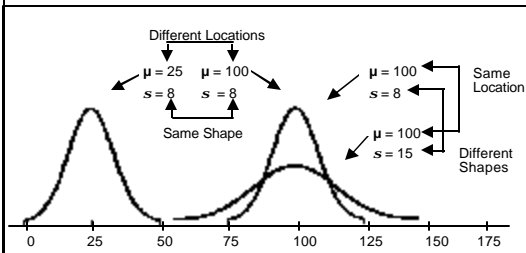
Shaded Area = $L \cdot W = 4 \cdot 0.2 = 0.8$



Heights of Adult Men and Women



Normal Distributions

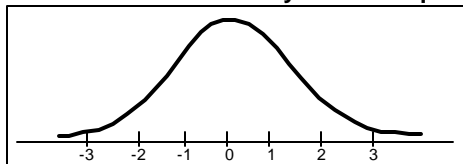


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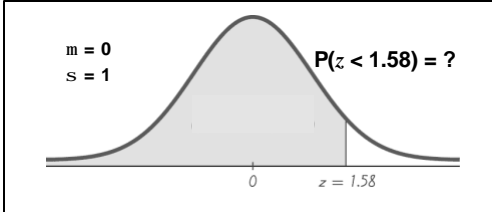
Definition

Standard Normal Distribution

a normal probability distribution that has a mean of 0 and a standard deviation of 1, and the total area under its density curve is equal to 1.



Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.



Methods for Finding Normal Distribution Areas

- ❖ Table A-2 (sometimes called z-distribution or z-table)
- ❖ STATDISK
- ❖ Other computer programs
- ❖ TI-83/84 Calculator

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Table A-2

- ❖ Formulas and Tables card
- ❖ Appendix

Table A-2
Standard Normal Distribution

Positive z-scores: cumulative from left

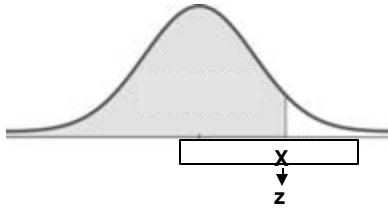
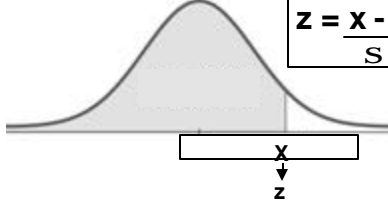


Table A-2
Standard Normal Distribution

$\mu = 0$

$\sigma = 1$

$$z = \frac{x - \mu}{\sigma}$$



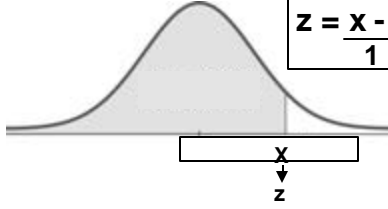
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Table A-2
Standard Normal Distribution

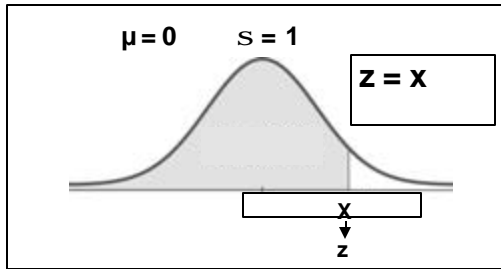
$\mu = 0$

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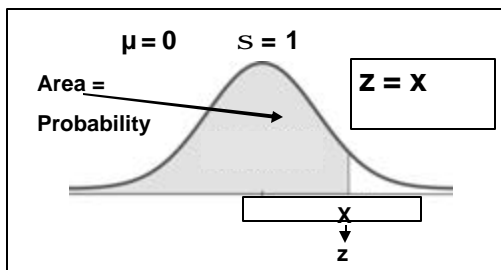
$$z = \frac{x - 0}{1}$$



**Table A-2
Standard Normal Distribution**



**Table A-2
Standard Normal Distribution**



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To find:

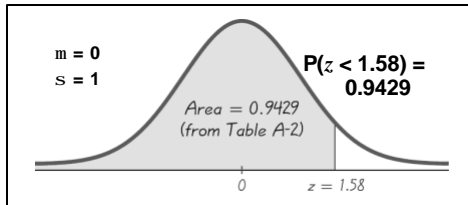
Z Score

the distance along horizontal scale of the standard normal distribution; refer to the leftmost column and top row of Table A-2

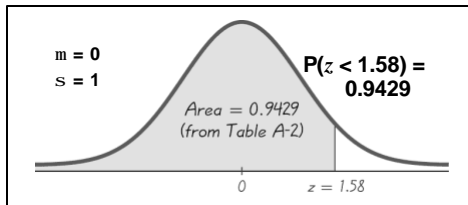
Area

the region under the curve; refer to the values in the body of Table A-2

Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.



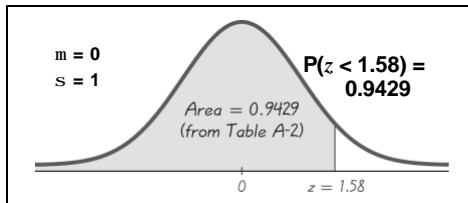
Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.



The probability that the chosen thermometer will measure freezing water less than 1.58 degrees is 0.9429.

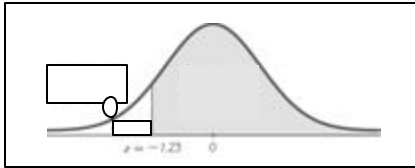
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Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.

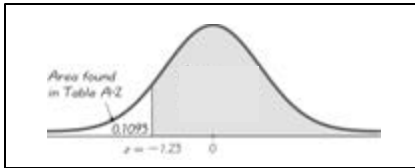


94.29% of the thermometers will read freezing water less than 1.58 degrees.

Example: If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above -1.23 degrees. $P(z > -1.23) = ?$

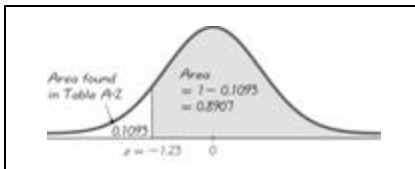


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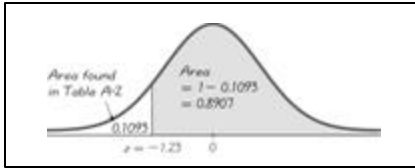


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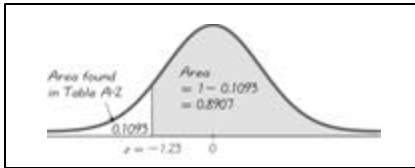
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Example: If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above -1.23 degrees. $P(z > -1.23) = 0.8907$



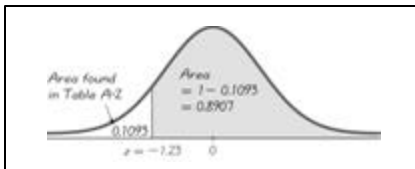
Example: If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above -1.23 degrees. $P(z > -1.23) = 0.8907$



The probability that the chosen thermometer with a reading above -1.23 degrees is 0.8907.

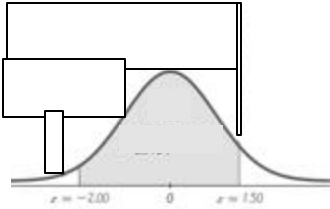
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Example: If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above -1.23 degrees. $P(z > -1.23) = 0.8907$

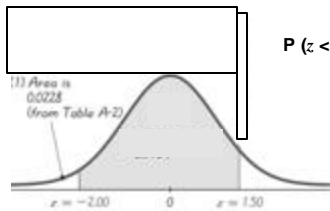


The percentage of thermometers with a reading above -1.23 degrees is 89.07%.

Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between -2.00 and 1.50 degrees.



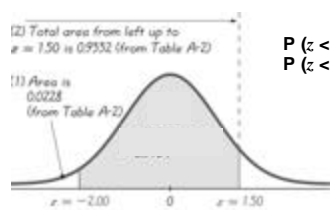
Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between -2.00 and 1.50 degrees.



$$P(z < -2.00) = 0.0228$$

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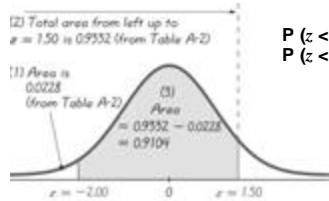
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$$P(z < -2.00) = 0.0228$$

$$P(z < 1.50) = 0.9332$$

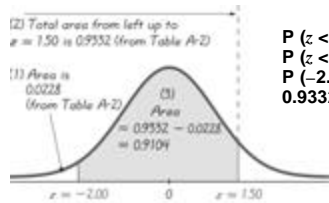
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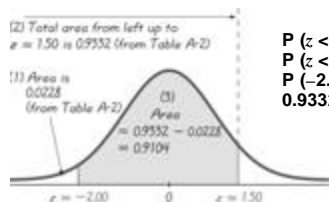
$$P(z < -2.00) = 0.0228$$

$$P(z < 1.50) = 0.9332$$

$$P(-2.00 < z < 1.50) = 0.9332 - 0.0228 = 0.9104$$

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Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between -2.00 and 1.50 degrees.



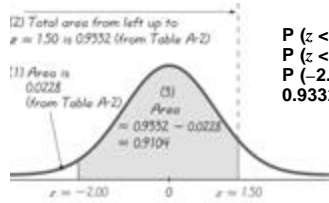
$$P(z < -2.00) = 0.0228$$

$$P(z < 1.50) = 0.9332$$

$$P(-2.00 < z < 1.50) = 0.9332 - 0.0228 = 0.9104$$

The probability that the chosen thermometer has a reading between -2.00 and 1.50 degrees is 0.9104 .

Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between -2.00 and 1.50 degrees.



$$P(z < -2.00) = 0.0228$$

$$P(z < 1.50) = 0.9332$$

$$P(-2.00 < z < 1.50) = 0.9332 - 0.0228 = 0.9104$$

The percentage of thermometers that read between -2.00 and 1.50 degrees is 91.04%.

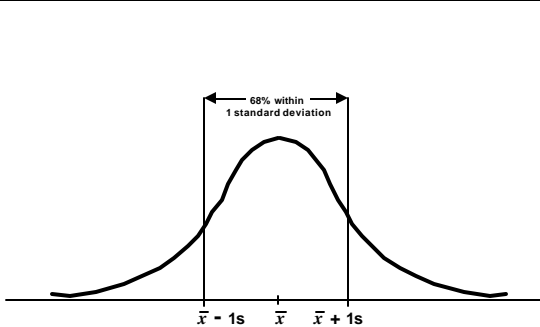
The Empirical Rule

Standard Normal Distribution: $\mu = 0$ and $\sigma = 1$

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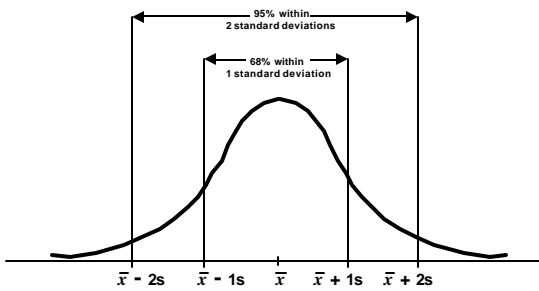
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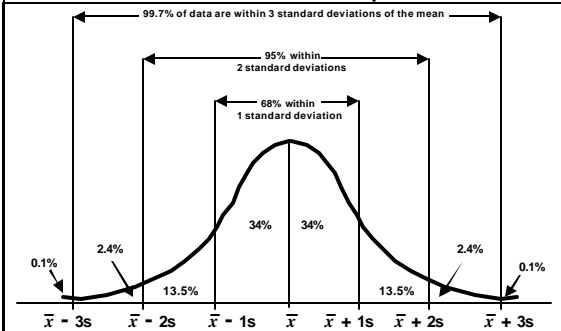


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The Empirical Rule

Standard Normal Distribution: $\mu = 0$ and $\sigma = 1$



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Notation

$$P(a < z < b)$$

denotes the probability that the z score is between a and b

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$$P(a < z < b)$$

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Notation

$$P(a < z < b)$$

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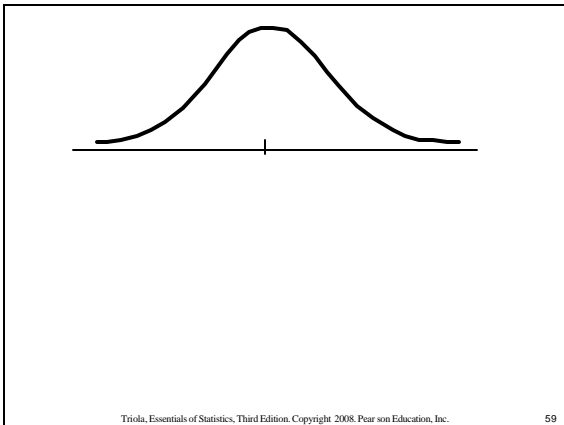
$$P(z > a)$$

greater than, at least, more than,
not less than

$$P(z < a)$$

less than, at most, no more than,
not greater than





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