6-5

The Central Limit Theorem

Definition

Sampling Distribution of the Mean
the probability distribution of sample means, with all samples having the same sample size *n*.
(In general, the sampling distribution of any statistic is the probability distribution of that statistic.)

Central Limit Theorem

Given:

1. The random variable x has a distribution (which may or may not be normal) with mean μ and standard deviation s.
2. Simple random samples all of the same size n are selected from the population. (The samples are selected so that all possible samples of size n have the same chance of being selected.)

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1. The distribution of sample means $\bar{x}$ will, as the sample size increases, approach a *normal* distribution.
2. The mean of the sample means will be the population mean µ.
3. The standard deviation of the sample means will approach $\sigma/\sqrt{n}$.

Practical Rules Commonly Used:

1. For samples of size n larger than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets better as the sample size n becomes larger.

2. If the original population is itself normally distributed, then the sample means will be normally distributed for <u>any sample size</u> n (not just the values of n larger than 30).

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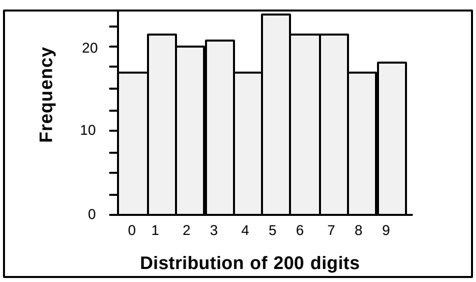
(often called standard error of the mean)

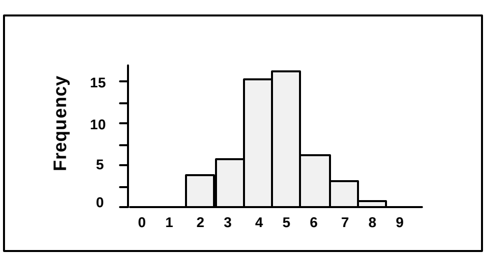
SSN digits				$\bar{x}$
1	8	6	4	4.75
5	3	3	6	4.25
9	8	8	8	8.25
5	1	2	5	3.25
9	3	3	5	5.00
4	2	6	2	3.50
7	7	1	6	5.25
9	1	5	4	4.75
5	3	3	9	5.00
7	3	4		
6	7	7	1	5.25
2	3	3	9	4.25
2	4	7	5	4.50
5	4	3	7	4.75
0	4	3	8	3.75
2	5	8	6	5.25
7	1	3	4	3.75
8	3	7	0	4.50
5	6	6	7	6.00

Distribution of 200 digits from Social Security Numbers

(Last 4 digits from 50 students)



Distribution of 50 Sample Means for 50 Students



As the sample size increases, the sampling distribution of sample means approaches a *normal* distribution.

Example: **Given the population of men has normally distributed weights with a mean of 172 lb and a standard deviation of 29 lb,**

a) if one man is randomly selected, find the probability that his weight is greater than 167 lb.

b) if 12 different men are randomly selected, find the probability that their mean weight is greater than 167 lb.

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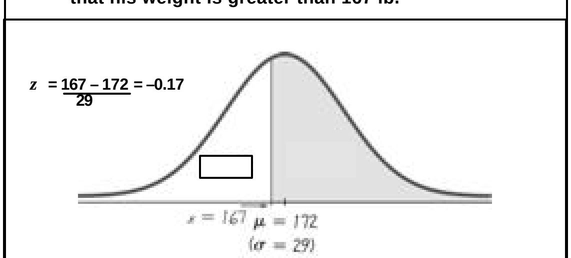
a) if one man is randomly selected, find the probability that his weight is greater than 167 lb.



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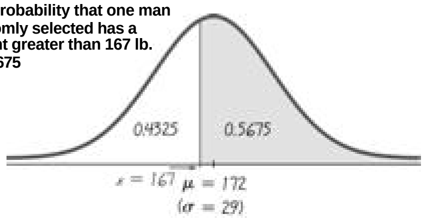
a) if one man is randomly selected, find the probability that his weight is greater than 167 lb.

The probability that one man randomly selected has a weight greater than 167 lb. is 0.5675



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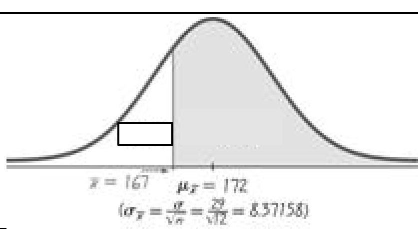

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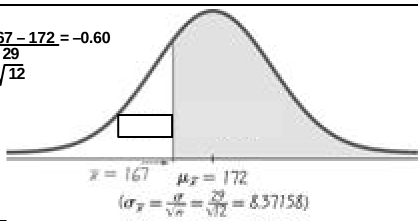
$$z = \frac{167 - 172}{\frac{29}{\sqrt{12}}} = -0.60$$



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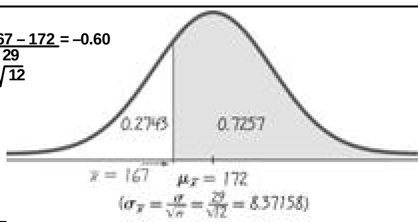

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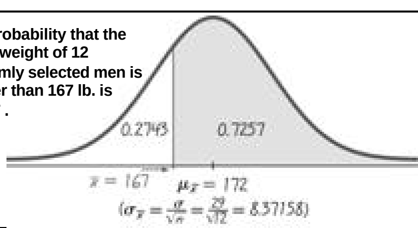
b.) if 12 different men are randomly selected, find the probability that their mean weight is greater than 167 lb.

The probability that the mean weight of 12 randomly selected men is greater than 167 lb. is 0.7257 .



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It is much easier for an individual to deviate from the mean than it is for a group of 12 to deviate from the mean.

Sampling Without Replacement

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finite population correction factor

Example: **IQ scores are normally distributed and have a mean of 100 and a standard deviation of 15. If 8 people are randomly selected, find the probability that their mean is at least 109.**