Exam #3 Review

Chapter 6
Normal Probability Distributions

6-2

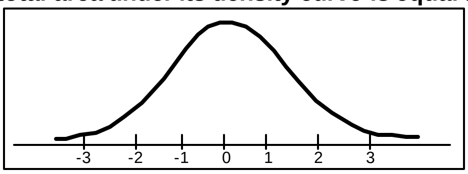
The Standard Normal Distribution

Because the total area under the density curve is equal to 1, there is a correspondence between area and probability.

Definition

Standard Normal Distribution

a normal probability distribution that has a mean of 0 and a standard deviation of 1, and the total area under its density curve is equal to 1.



Example: **If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.**

$P(z > -1.23) = ?$

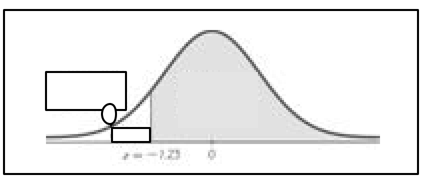


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Table A-2

- ❖ **Formulas and Tables card**
- ❖ **Appendix**

NEGATIVE Z Scores   Table A-2

TABLE A-2 Standard Normal (*z*) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

Table A-2

- **Designed only for *standard* normal distribution**
- **Is on two pages: *negative* z-scores and *positive* z-scores**
- **Body of table is a cumulative area from the left up to a vertical boundary**
- **Avoid confusion between z-scores and areas**
- **Z-score hundredths is across the top row**

Table A-2
Standard Normal Distribution

Negative z-scores: cumulative from left

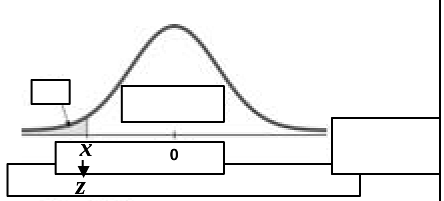


Table A-2
Standard Normal Distribution

Positive z-scores: cumulative from left

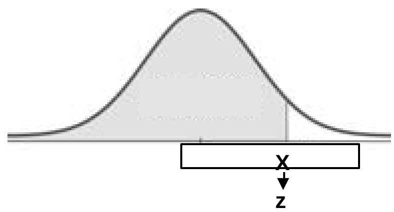


Table A-2
Standard Normal Distribution

μ = 0   s = 1

$$z = \frac{x - 0}{1}$$

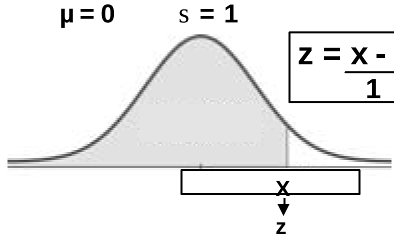
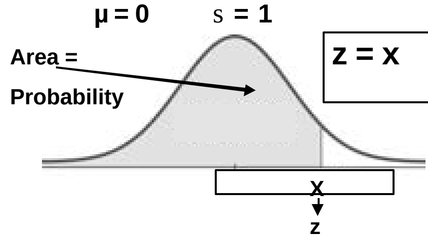


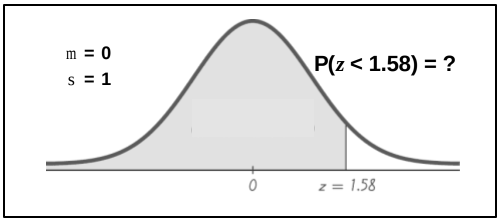
Table A-2
Standard Normal Distribution

μ = 0   s = 1

$$z = x$$



Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.

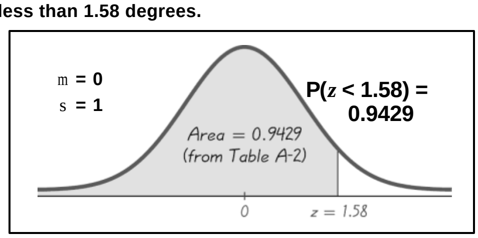


POSITIVE z Scores

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	* .9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890

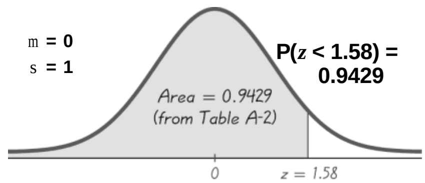
Example: If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.



The probability that the chosen thermometer will measure freezing water less than 1.58 degrees is 0.9429.

Example: **If thermometers have an average (mean) reading of 0 degrees and a standard deviation of 1 degree for freezing water and if one thermometer is randomly selected, find the probability that it reads freezing water is less than 1.58 degrees.**

m = 0
s = 1

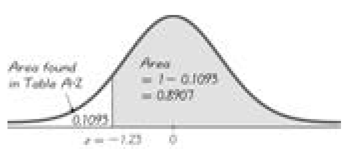


P(z < 1.58) = 0.9429

94.29% of the thermometers will read freezing water less than 1.58 degrees.

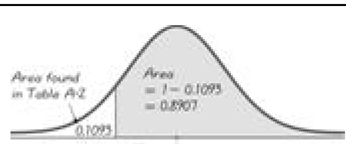
Example: **If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.**

P (z > –1.23) = ?



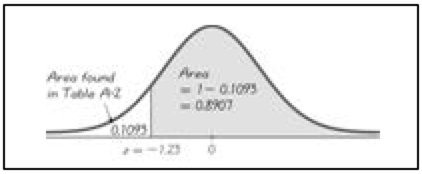
Example: **If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.**

P (z > –1.23) = 0.8907



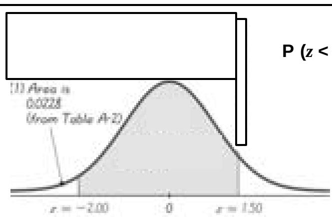
Example: If we are using the same thermometers, and if one thermometer is randomly selected, find the probability that it reads (at the freezing point of water) above –1.23 degrees.

P (z > –1.23) = 0.8907



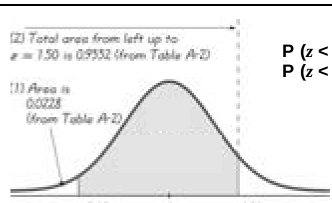
The percentage of thermometers with a reading above -1.23 degrees is 89.07%.

Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.



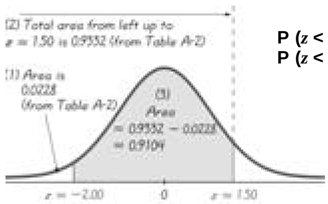
P (z < –2.00) = 0.0228

Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.



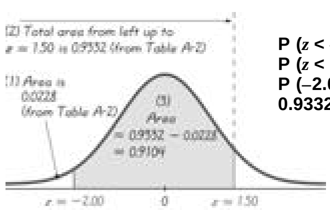
P (z < –2.00) = 0.0228
P (z < 1.50) = 0.9332

Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.



P ($z < -2.00$) = 0.0228
P ($z < 1.50$) = 0.9332

Example: A thermometer is randomly selected. Find the probability that it reads (at the freezing point of water) between –2.00 and 1.50 degrees.

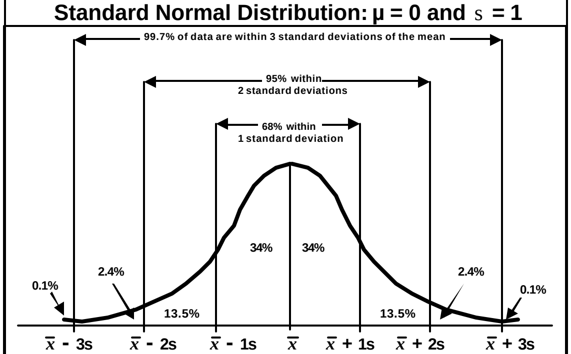


P ($z < -2.00$) = 0.0228
P ($z < 1.50$) = 0.9332
P ($-2.00 < z < 1.50$) =
0.9332 – 0.0228 = 0.9104

The probability that the chosen thermometer has a reading between - 2.00 and 1.50 degrees is 0.9104.

The Empirical Rule

Standard Normal Distribution: μ = 0 and s = 1



Notation

P(a < z < b)
denotes the probability that the z score is between a and b

P(z > a)
denotes the probability that the z score is greater than a

P (z < a)
denotes the probability that the z score is less than a

Notation

P(a < z < b)
between a and b

P(z > a)
greater than, at least, more than, not less than

P (z < a)
less than, at most, no more than, not greater than

6-3

Applications of Normal Distributions

Converting to Standard Normal Distribution

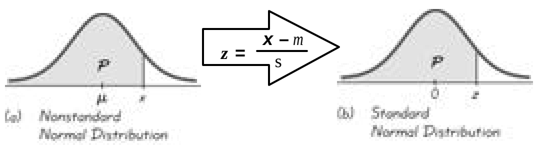
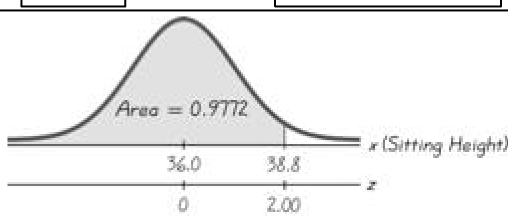


Figure 6-12

Probability of Sitting Heights Less Than 38.8 Inches

$m = 36.0$
$s = 1.4$

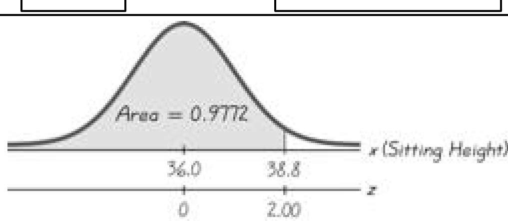
$$z = \frac{38.8 - 36.0}{1.4} = 2.00$$



Probability of Sitting Heights Less Than 38.8 Inches

$m = 36.0$
$s = 1.4$

P (x < 38.8 in.) = P(z < 2)
= 0.9772



6.2 – 6.3

Finding Values of Normal Distributions

Cautions to keep in mind

1. **Don't confuse z scores and areas.**

   z scores are distances along the horizontal scale, but areas are regions under the normal curve.

   Table A-2 lists z scores in the left column and across the top row, but areas are found in the body of the table.

2. **Choose the correct (right/left) side of the graph.**

3. **A z score must be negative whenever it is located to the left half of the normal distribution.**

4. **Areas (or probabilities) are positive or zero values, but they are never negative.**

Procedure for Finding Values Using Table A-2 and Formula 6-2

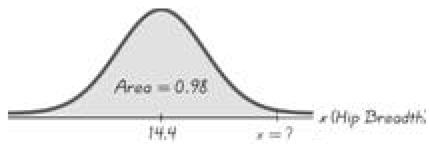
1. **Sketch a normal distribution curve, enter the given probability or percentage in the appropriate region of the graph, and identify the x value(s) being sought.**
2. **Use Table A-2 to find the z score corresponding to the cumulative left area bounded by x. Refer to the BODY of Table A-2 to find the closest area, then identify the corresponding z score.**
3. **Using Formula 6-2, enter the values for μ, s, and the z score found in step 2, then solve for x.**

   $$x = \mu + (z \cdot s)$$ **(another form of Formula 6-2)**

(If z is located to the left of the mean, be sure that it is a negative number.)

4. **Refer to the sketch of the curve to verify that the solution makes sense in the context of the graph and the context of the problem.**

Find P_{98} for Hip Breadths of Men



Find P_{98} for Hip Breadths of Men

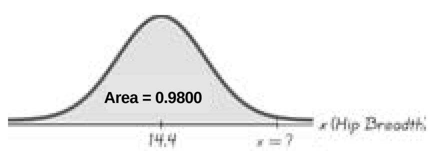


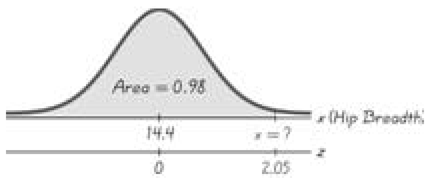
Table A-2: Positive Z- scores

TABLE A-2 (*continued*) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	[illegible]	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495 *	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890

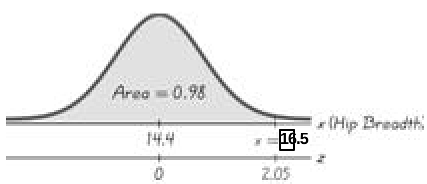
Find P_{98} for Hip Breadths of Men

$x = \mu + (z \cdot \sigma)$
$x = 14.4 + (2.05 \cdot 1.0)$
$x = 16.45$



Find P_{98} for Hip Breadths of Men

The hip breadth of 16.5 in. separates the lowest 98% from the highest 2%



6-5

The Central Limit Theorem

Definition

Sampling Distribution of the Mean
the probability distribution of sample means, with all samples having the same sample size n.
(In general, the sampling distribution of any statistic is the probability distribution of that statistic.)

Central Limit Theorem

Given:

1. **The random variable x has a distribution (which may or may not be normal) with mean μ and standard deviation s.**
2. **Simple random samples all of the same size *n* are selected from the population. (The samples are selected so that all possible samples of size *n* have the same chance of being selected.)**

Central Limit Theorem

Conclusions:

1. **The distribution of sample means $\bar{x}$ will, as the sample size increases, approach a *normal* distribution.**
2. **The mean of the sample means will be the population mean µ.**
3. **The standard deviation of the sample means will approach $s/\sqrt{n}$.**

Practical Rules Commonly Used:

1. For samples of size n larger than 30, the distribution of the sample means can be approximated reasonably well by a normal distribution. The approximation gets better as the sample size n becomes larger.

2. If the original population is itself normally distributed, then the sample means will be normally distributed for <u>any sample size</u> n (not just the values of n larger than 30).

Notation

the mean of the sample means

$$\mu_{\bar{x}} = \mu$$

the standard deviation of sample means

$$s_{\bar{x}} = \frac{s}{\sqrt{n}}$$

(often called standard error of the mean)

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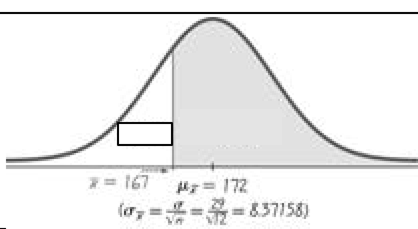
Example: Given the population of men has normally distributed weights with a mean of 172 lb. and a standard deviation of 29 lb,
b.) if 12 different men are randomly selected, find the probability that their mean weight is greater than 167 lb.



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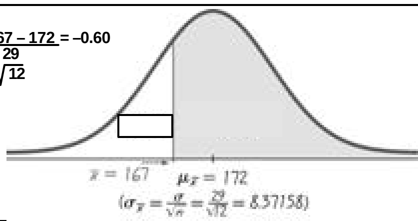
Example: **Given the population of men has normally distributed weights with a mean of 172 lb. and a standard deviation of 29 lb,**
b.) if 12 different men are randomly selected, find the probability that their mean weight is greater than 167 lb.

$$z = \frac{167 - 172}{\frac{29}{\sqrt{12}}} = -0.60$$



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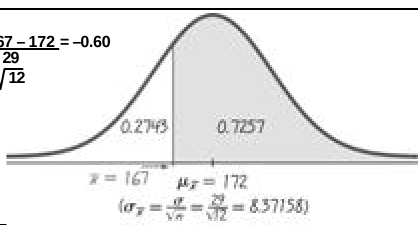
Example: **Given the population of men has normally distributed weights with a mean of 172 lb. and a standard deviation of 29 lb,**
b.) if 12 different men are randomly selected, find the probability that their mean weight is greater than 167 lb.

$$z = \frac{167 - 172}{\frac{29}{\sqrt{12}}} = -0.60$$



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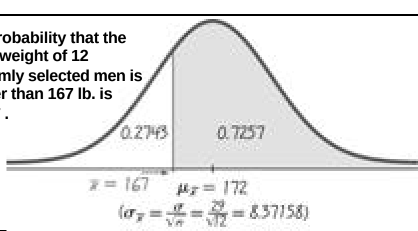
Example: **Given the population of men has normally distributed weights with a mean of 172 lb. and a standard deviation of 29 lb,**
b.) if 12 different men are randomly selected, find the probability that their mean weight is greater than 167 lb.

The probability that the mean weight of 12 randomly selected men is greater than 167 lb. is 0.7257 .



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