

6.2 – 6.3

Finding Values of Normal Distributions

Cautions to keep in mind

1. Don't confuse z scores and areas.

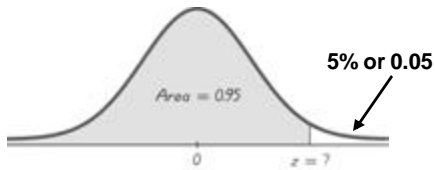
Z scores are distances along the horizontal scale, but areas are regions under the normal curve.

Table A2 lists z scores in the left column and across the top row, but areas are found in the body of the table.

2. Choose the correct (right/left) side of the graph.
3. A z score must be negative whenever it is located to the left half of the normal distribution.
4. Areas (or probabilities) are positive or zero values, but they are never negative.

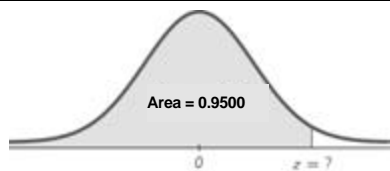
Finding z Scores When Given Percentages or Probabilities

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Find the z -score of the P_{95}
(z score will be positive)

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Table A-2: Positive Z- scores

TABLE A-2 (continued) Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890

**Procedure for Finding Values
Using Table A-2 and Formula 6-2**

1. Sketch a normal distribution curve, enter the given probability or percentage in the appropriate region of the graph, and identify the x value(s) being sought.
2. Use Table A-2 to find the z score corresponding to the cumulative left area bounded by x . Refer to the BODY of Table A-2 to find the closest area, then identify the corresponding z score.
3. Using Formula 6-2, enter the values for μ , S , and the z score found in step 2, then solve for x .

$$x = \mu + (z \cdot S) \text{ (another form of Formula 6-2)}$$

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(If z is located to the left of the mean, be sure that it is a negative number.)

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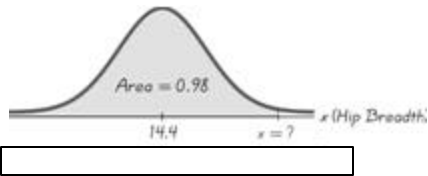
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4. Refer to the sketch of the curve to verify that the solution makes sense in the context of the graph and the context of the problem.

Example

In designing seats to be installed in commercial aircraft, engineers want to make the seats wide enough to fit 98% of all males. Men have hip breadths that are normally distributed with a mean of 14.4 in. and a standard deviation of 1.0 in. Find P_{98} ; that is, find the hip breadth of the men that separated the bottom 98% from the top 2%.

Find P_{98} for Hip Breadths of Men



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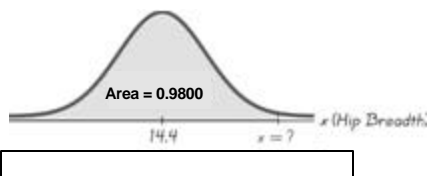


Table A-2: Positive Z- scores

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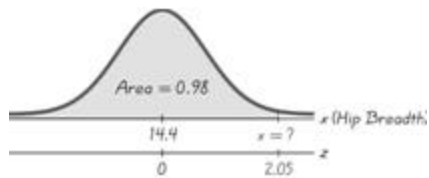
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2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890

Find P_{98} for Hip Breadths of Men

$$z = 2.05$$

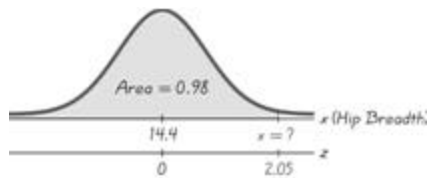


Find P_{98} for Hip Breadths of Men

$$x = m + (z \cdot s)$$

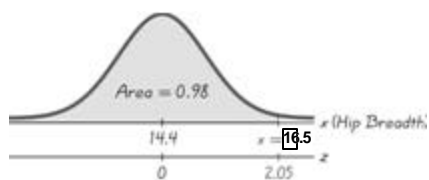
$$x = 14.4 + (2.05 \cdot 1.0)$$

$$x = 16.45$$



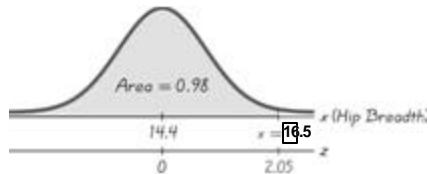
Find P_{98} for Hip Breadths of Men

The hip breadth of 16.5 in. separates the lowest 98% from the highest 2%



Find P_{98} for Hip Breadths of Men

Seats designed for a hip breadth up to 16.5 in. will fit 98% of men.



Example

When designing the placement of a CD player in a new model car, engineers must consider the forward grip reach of the driver. It was decided that the CD should be placed so that it is within the forward grip reach of 95% of women. Women have forward grip reaches that are normally distributed with a mean of 27.0 in. and a standard deviation of 1.3 in. Find the forward grip reach of women that separates the longest 95% from the others.

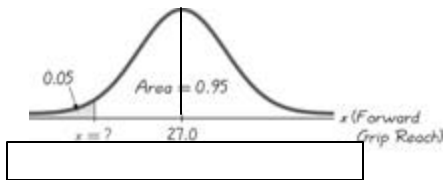
Example

When designing the placement of a CD player in a new model car, engineers must consider the forward grip reach of the driver. It was decided that the CD should be placed so that it is within the forward grip reach of 95% of women. Women have forward grip reaches that are normally distributed with a mean of 27.0 in. and a standard deviation of 1.3 in. Find the forward grip reach of women that separates the longest 95% from the shortest 5%.

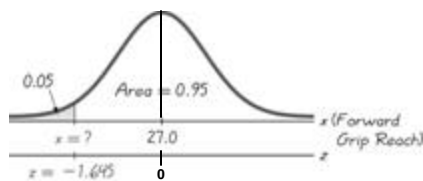
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Finding $P_{.05}$ for Grips of Women

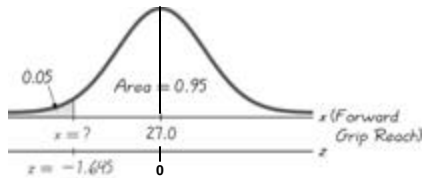


Finding $P_{.05}$ for Grips of Women



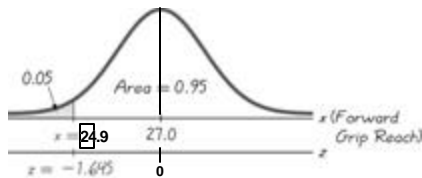
Finding P_{05} for Grips of Women

$$x = 27.0 + (-1.645 \cdot 1.3) = 24.8615$$



Finding P_{05} for Grips of Women

The forward grip of 24.9 in. (rounded) separates the top 95% from the others.

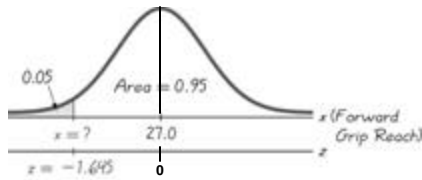


REMEMBER!

The z score will be negative if the value is located to the left (below) the mean. Otherwise, the z score will be positive.

Finding P_{05} for Grips of Women Mistake!!

$$x = 27.0 + (1.645 \cdot 1.3) = 29.1385$$



Finding P_{05} for Grips of Women Mistake!!

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