

# Chapter 8

## Hypothesis Testing

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# Chapter 8

## Hypothesis Testing

- 8-1 Overview
- 8-2 Basics of Hypothesis Testing
- 8-3 Testing a Claim About a Proportion
- 8-5 Testing a Claim About a Mean:  $\sigma$  Not Known
- 8-6 Testing a Claim About a Standard Deviation or Variance

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# 8-1 Overview

## Definition

### ❖ Hypothesis

in statistics, is a claim or statement about a property of a population

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## 8-1 Overview

### Definition

#### ❖ Hypothesis Test

is a standard procedure for testing a claim about a property of a population

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### Rare Event Rule for Inferential Statistics

If, under a given assumption, the probability of a particular observed event is exceptionally small, we conclude that the assumption is probably not correct.

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**Example:** ProCare Industries, Ltd., once provided a product called “Gender Choice,” which, according to advertising claims, allowed couples to “increase your chances of having a boy up to 85%, a girl up to 80%.” Suppose we conduct an experiment with 100 couples who want to have baby girls, and they all follow the Gender Choice directions in the pink package. For the purpose of testing the claim of an increased likelihood for girls, we will assume that Gender Choice has no effect. Using common sense and no formal statistical methods, what should we conclude about the assumption of no effect from Gender Choice if 100 couples using Gender Choice have 100 babies consisting of

a) 52 girls?; b) 97 girls?

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**Example: ProCare Industries, Ltd.: Part a)**

a) We normally expect around 50 girls in 100 births. The results of 52 girls is close to 50, so we should not conclude that the Gender Choice product is effective. If the 100 couples used no special method of gender selection, the result of 52 girls could easily occur by chance. The assumption of no effect from Gender Choice appears to be correct. There isn't sufficient evidence to say that Gender Choice is effective.

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**Example: ProCare Industries, Ltd.: Part b)**

b) The result of 97 girls in 100 births is extremely unlikely to occur by chance. We could explain the occurrence of 97 girls in one of two ways: Either an *extremely rare* event has occurred by chance, or Gender Choice is effective. The extremely low probability of getting 97 girls is strong evidence against the assumption that Gender Choice has no effect. It does appear to be effective.

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**8-2**

**Basics of Hypothesis Testing**

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## 8-2 Section Objectives

- ❖ Given a claim, identify the null hypothesis, and the alternative hypothesis, and express them both in symbolic form.
- ❖ Given a claim and sample data, calculate the value of the test statistic.
- ❖ Given a significance level, identify the critical value(s).
- ❖ Given a value of the test statistic, identify the  $P$ -value.
- ❖ State the conclusion of a hypothesis test in simple, non-technical terms.
- ❖ Identify the type I and type II errors that could be made when testing a given claim.

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**Example:** Let's again refer to the Gender Choice product that was once distributed by ProCare Industries. ProCare Industries claimed that couple using the pink packages of Gender Choice would have girls at a rate that is greater than 50% or 0.5. Let's again consider an experiment whereby 100 couples use Gender Choice in an attempt to have a baby girl; let's assume that the 100 babies include exactly 52 girls, and let's formalize some of the analysis.

Under normal circumstances the proportion of girls is 0.5, so a claim that Gender Choice is effective can be expressed as  $p > 0.5$ .

Using a normal distribution as an approximation to the binomial distribution, we find  $P(52 \text{ or more girls in } 100 \text{ births}) = 0.3821$ .

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**Example:** Let's again refer to the Gender Choice product that was once distributed by ProCare Industries. ProCare Industries claimed that couple using the pink packages of Gender Choice would have girls at a rate that is greater than 50% or 0.5. Let's again consider an experiment whereby 100 couples use Gender Choice in an attempt to have a baby girl; let's assume that the 100 babies include exactly 52 girls, and let's formalize some of the analysis.

Figure 8-1 shows that with a probability of 0.5, the outcome of 52 girls in 100 births is not unusual.

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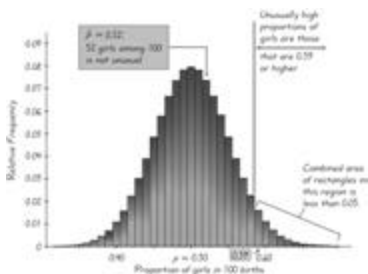


Figure 8-1

We do not reject random chance as a reasonable explanation. We conclude that the proportion of girls born to couples using Gender Choice is not significantly greater than the number that we would expect by random chance.

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## Key Points

- ❖ **Claim:** For couples using Gender Choice, the proportion of girls is  $p > 0.5$ .
- ❖ **Working assumption:** The proportion of girls is  $p = 0.5$  (with no effect from Gender Choice).
- ❖ **The sample resulted in 52 girls among 100 births, so the sample proportion is  $\hat{p} = 52/100 = 0.52$ .**

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## Key Points

- ❖ **Assuming that  $p = 0.5$ , we use a normal distribution as an approximation to the binomial distribution to find that  $P(\text{at least 52 girls in 100 births}) = 0.3821$ .**
- ❖ **There are two possible explanation for the result of 52 girls in 100 births: Either a random chance event (with probability 0.3821) has occurred, or the proportion of girls born to couples using Gender Choice is greater than 0.5.**
- ❖ **There isn't sufficient evidence to support Gender Choice's claim.**

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# Components of a Formal Hypothesis Test

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## Null Hypothesis: $H_0$

- ❖ Statement about value of population parameter that is equal to some claimed value

$$H_0: p = 0.5 \quad H_0: m = 98.6 \quad H_0: s = 15$$

- ❖ Test the Null Hypothesis directly
- ❖ Reject  $H_0$  or fail to reject  $H_0$

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## Alternative Hypothesis: $H_1$

- ❖ the statement that the parameter has a value that somehow differs from the null
- ❖ Must be true if  $H_0$  is false
- ❖  $^1, <, >$

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**Claim:** Using math symbols

**$H_0$ :** Must contain equality

**$H_1$ :** Will contain <sup>1</sup>, <, >

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### Note about Identifying $H_0$ and $H_1$



Figure 8-2

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### Note about Forming Your Own Claims (Hypotheses)

If you are conducting a study and want to use a hypothesis test to support your claim, the claim must be worded so that it becomes the alternative hypothesis. This means your claim must be expressed using only <sup>1</sup>, <, >

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## Test Statistic

The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Test statistic for proportions

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## Test Statistic

The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

$$z = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}}$$

Test statistic for mean

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## Test Statistic

The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}}$$

Test statistic for mean

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## Test Statistic

The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

$$C^2 = \frac{(n-1)s^2}{S^2}$$

Test statistic for standard deviation

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**Example:** A survey of  $n = 880$  randomly selected adult drivers showed that 56% (or  $\hat{p} = 0.56$ ) of those respondents admitted to running red lights. Find the value of the test statistic for the claim that the majority of all adult drivers admit to running red lights. (In Section 8-3 we will see that there are assumptions that must be verified. For this example, assume that the required assumptions are satisfied and focus on finding the indicated test statistic.)

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**Solution:** The preceding example showed that the given claim results in the following null and alternative hypotheses:  $H_0: p = 0.5$  and  $H_1: p > 0.5$ . Because we work under the assumption that the null hypothesis is true with  $p = 0.5$ , we get the following test statistic:

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.56 - 0.5}{\sqrt{\frac{(0.5)(0.5)}{880}}} = 3.56$$

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**Interpretation:** We know from previous chapters that a  $z$  score of 3.56 is exceptionally large. It appears that in addition to being "more than half," the sample result of 56% is *significantly* more than 50%.

We could show that the sample proportion of 0.56 (from 56%) does fall within the range of values considered to be significant because they are so far above 0.5 that they are not likely to occur by chance (assuming that the population proportion is  $p = 0.5$ ).

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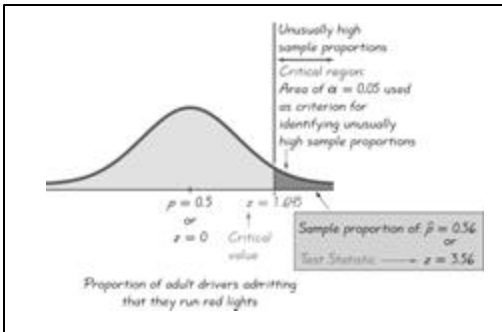
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### Critical Region (or Rejection Region)

Set of all values of the test statistic that would cause a rejection of the null hypothesis

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## Critical Region

Set of all values of the test statistic that would cause a rejection of the null hypothesis



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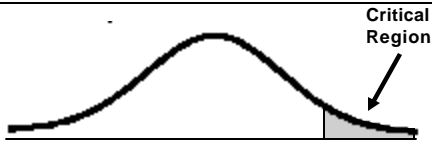
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## Critical Region

Set of all values of the test statistic that would cause a rejection of the null hypothesis



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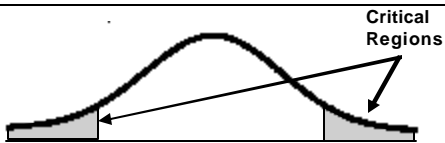
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## Critical Region

Set of all values of the test statistic that would cause a rejection of the null hypothesis



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## Significance Level

- ❖ denoted by  $\alpha$
- ❖ the probability that the test statistic will fall in the critical region when the null hypothesis is actually true.
- ❖ same  $\alpha$  introduced in Section 7-2
- ❖ common choices are 0.05, 0.01, and 0.10

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## Critical Value

Any value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to a rejection of the null hypothesis

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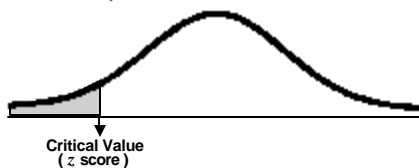
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## Critical Value

Any value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to a rejection of the null hypothesis



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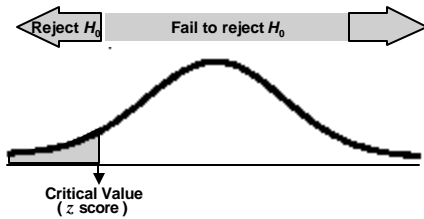
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## Critical Value

Any value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to a rejection of the null hypothesis




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## Two-tailed, Right-tailed, Left-tailed Tests

The tails in a distribution are the extreme regions bounded by critical values.

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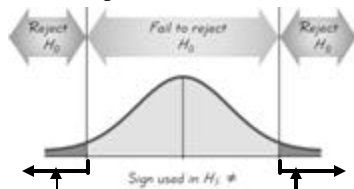
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## Two-tailed Test

$H_0: =$   $\alpha$  is divided equally between the two tails of the critical region  
 $H_1: \neq$

Means less than or greater than



Values that differ significantly from  $H_0$

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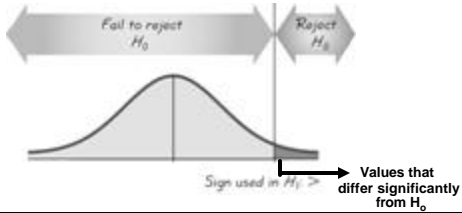
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## Right-tailed Test

$$H_0: =$$

$$H_1: >$$



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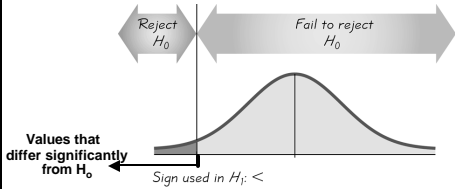
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## Left-tailed Test

$$H_0: =$$

$$H_1: <$$



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## P-Value

The *P*-value (or *p*-value or probability value) is the probability of getting a value of the test statistic that is *at least as extreme* as the one representing the sample data, assuming that the null hypothesis is true. The null hypothesis is rejected if the *P*-value is very small, such as 0.05 or less.

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### Example: Finding P-values

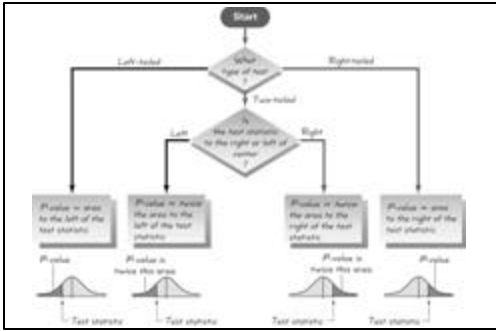


Figure 8-6

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### Conclusions in Hypothesis Testing

❖ always test the null hypothesis

1. Reject the  $H_0$
2. Fail to reject the  $H_0$

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### Decision Criterion

Traditional method:

***Reject  $H_0$***  if the test statistic falls within the critical region.

***Fail to reject  $H_0$***  if the test statistic does not fall within the critical region.

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## Decision Criterion

**P-value method:**

**Reject  $H_0$  if  $P\text{-value} \leq \alpha$  (where  $\alpha$  is the significance level, such as 0.05).**

**Fail to reject  $H_0$  if  $P\text{-value} > \alpha$ .**

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## Decision Criterion

**Another option:**

**Instead of using a significance level such as 0.05, simply identify the  $P$ -value and leave the decision to the reader.**

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## Decision Criterion

**Confidence Intervals:**

**Because a confidence interval estimate of a population parameter contains the likely values of that parameter, reject a claim that the population parameter has a value that is not included in the confidence interval.**

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# Wording the Final Conclusion

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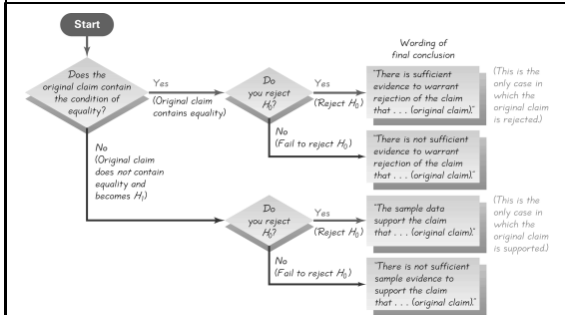
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## Wording of Final Conclusion



**Figure 8-7**

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## Accept versus Fail to Reject

- ❖ Some texts use “accept the null hypothesis
- ❖ We are not proving the null hypothesis
- ❖ Sample evidence is not strong enough to warrant rejection (such as not enough evidence to convict a suspect)

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## Type I Error

- ❖ A Type I error is the mistake of rejecting the null hypothesis when it is true.
- ❖ The symbol  $\alpha$  (alpha) is used to represent the probability of a type I error.

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## Type II Error

- ❖ A Type II error is the mistake of failing to reject the null hypothesis when it is false.
- ❖ The symbol  $\beta$  (beta) is used to represent the probability of a type II error.

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Table 7-1 Type I and Type II Errors

		True State of Nature	
		The null hypothesis is true.	The null hypothesis is false.
Decision	We decide to reject the null hypothesis.	Type I error (rejecting a true null hypothesis) $\alpha$	Correct decision
	We fail to reject the null hypothesis.	Correct decision	Type II error (failing to reject a false null hypothesis) $\beta$

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## Controlling Type I and Type II Errors

- ❖ For any fixed  $a$ , an increase in the sample size  $n$  will cause a decrease in  $b$ .
- ❖ For any fixed sample size  $n$ , a decrease in  $a$  will cause an increase in  $b$ . Conversely, an increase in  $a$  will cause a decrease in  $b$ .
- ❖ To decrease both  $a$  and  $b$ , increase the sample size.

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## Definition

### Power of a Hypothesis Test

The power of a hypothesis test is the probability  $(1 - b)$  of rejecting a false null hypothesis, which is computed by using a particular significance level  $a$  and a particular value of the population parameter that is an alternative to the value assumed true in the null hypothesis.

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#### Traditional Method

1. Identify the specific claim or hypotheses to be tested and put it in symbolic form.
2. Write the symbolic form that must be true when the original claim is false.
3. Write the symbolic expression for the null hypothesis and the alternative hypothesis. The null hypothesis is the symbolic expression that the parameter equals the stated value being considered.
4. Select the significance level  $\alpha$  based on the conditions of the test. If error is made at either of the consequences of rejecting  $H_0$ , use  $\alpha/2$  for each.
5. Identify the statistic that is used in the test and determine the sampling distribution for the critical value.
6. Find the test statistic, the critical values, and the critical region. There is a great need to write the test statistic in terms of the null and alternative hypotheses.
7. Reject  $H_0$  if the test statistic is in the critical region.
8. Fail to reject  $H_0$  if the test statistic is not in the critical region.
9. Revise the original claim or hypotheses, if necessary, and address the original claim.

## Comprehensive Hypothesis Test

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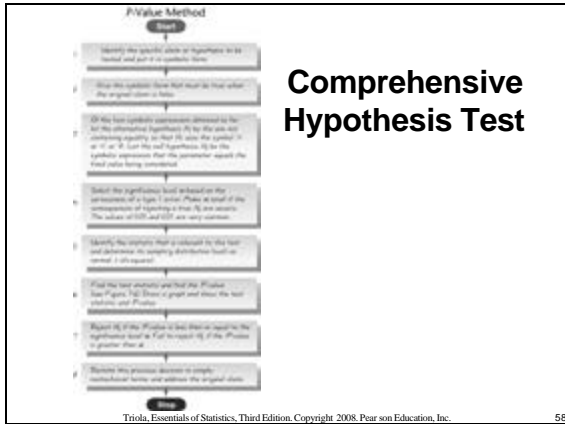
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## Comprehensive Hypothesis Test

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## Comprehensive Hypothesis Test

A confidence interval estimate of a population parameter contains the likely values of that parameter. We should therefore reject a claim that the population parameter has a value that is not included in the confidence interval.

**Confidence Interval Method**

Construct a confidence interval with a confidence level selected as in Table 7.2.

Because a confidence interval estimate of a population parameter contains the likely values of that parameter, reject a claim that the population parameter has a value that is not included in the confidence interval.

Significance Level for Hypothesis Test	Confidence Level for Confidence Interval	
	Two-Tailed Test	One-Tailed Test
0.05	95%	90%
0.10	90%	80%

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## Comprehensive Hypothesis Test

**Caution:** In some cases, a conclusion based on a confidence interval may be different from a conclusion based on a hypothesis test. See the comments in the individual sections.

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