8-3

Testing a Claim about a Proportion

Assumptions

for testing claims about population proportions

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3) The condition $np \geq 5$ and $nq \geq 5$ are satisfied, so the binomial distribution of sample proportions can be approximated by a normal distribution with $\mu = np$ and $\sigma = \sqrt{npq}$

Notation

n = number of trials

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$\hat{p}$ = x/n (*sample* proportion)

p = population proportion (used in the null hypothesis)

$q = 1 - p$

Test Statistic for Testing a Claim about a Proportion

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

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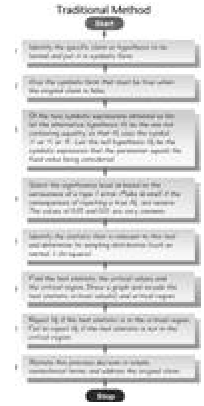
Critical values and P-values: Use Table A-2

CAUTION

When the calculation of $\hat{p}$ results in a decimal with many places, do not round too severely when evaluating the *z* test statistic.

Traditional Method

Same as described in Sections 8-2 and in Figure 8-9



Comprehensive Hypothesis Test

Example: **It was found that 821 crashes of midsize cars equipped with air bags, 46 of the crashes resulted in hospitalization of the drivers. Using the 0.01 significance level, test the claim that the air bag hospitalization is lower than the 7.8% rate for cars with automatic safety belts.**

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$np = (821)(0.078) > 5$

$nq = (821)(0.922) > 5$

Example: It was found that 821 crashes of midsize cars equipped with air bags, 46 of the crashes resulted in hospitalization of the drivers. Using the 0.01 significance level, test the claim that the air bag hospitalization is lower than the 7.8% rate for cars with automatic safety belts.

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H_0: $p = 0.078$
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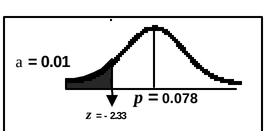

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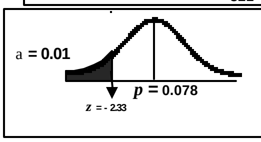

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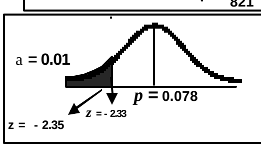

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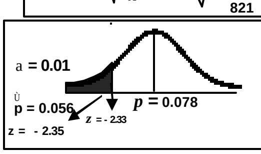

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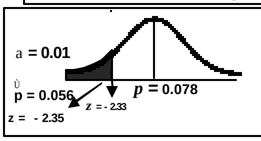

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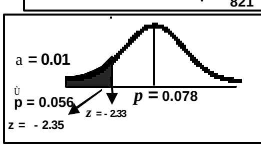

There is sufficient evidence to support claim that the air bag hospitalization rate is lower than the 7.8% rate for automatic safety belts.

P-Value Method

Same as described in Section 8-2 and Figure 8-8

Reject the null hypothesis if the *P*-value is less than or equal to the significance level α.

P-Value Method

Same as described in Section 8-2 and Figure 8-8

Test Statistic: -2.35

P-Value of Test Statistic: 0.0094

α = 0.01

0.0094 < 0.01

REJECT the H_o

Confidence Interval Method

Same as described in Section 8-2 and Table 8-2

Reject the null hypothesis if the confidence interval does not contain the population parameter.

Confidence Interval Method

Same as described in Section 8-2 and Table 8-2

If $\hat{p}$ = 0.0560 and α = 0.01,

the confidence interval would be:

0.0353 < p < 0.0767

Reject the claim since this interval does not contain the claimed proportion 0.078.

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$$\hat{p} = \frac{x}{n} = \frac{96}{(96+54)} = 0.64$$

(determining the sample proportion of households with cable TV)

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$$z = \frac{x - \mu}{\sigma} = \frac{x - np}{\sqrt{npq}} = \frac{\frac{x}{n} - \frac{np}{n}}{\frac{\sqrt{npq}}{n}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Example: **In a recent year, of the 109,857 arrests for Federal offenses, 29.1% were for drug offenses (based on data from the U.S. Department of Justice). Use the 0.01 significance level to test the claim that the drug offense rate is equal to 30%. How can the result be explained, given that the 29.1% appears to be so close to 30%.**