8-5

Testing a Claim about a Mean: σ Not Known

Assumptions

for testing claims about population means

1) The sample is a simple random sample.
2) The value of the population standard deviation σ is *not* known.
3) Either or both of these conditions are satisfied: The population is normally distributed or n > 30.

Test Statistic

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}}$$

Critical Values and P-Values

- Found in Table A-3
- Degrees of freedom (df) = n - 1

Important Properties of the Student *t* Distribution

1. The Student *t* distribution is different for different sample sizes (see Figure 7-5 in Section 7-4).
2. The Student *t* distribution has the same general bell shape as the normal distribution; its wider shape reflects the greater variability that is expected when *s* is used to estimate *s* .
3. The Student *t* distribution has a mean of $t = 0$ (just as the standard normal distribution has a mean of $z = 0$).
4. The standard deviation of the Student *t* distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has a *s* = 1).
5. As the sample size *n* gets larger, the Student *t* distribution get closer to the normal distribution.

Use the Student t distribution when *s* is not known and either or both of these conditions is satisfied:

The population is normally distributed

or

n > 30.

Example: Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.

270 273 258 204 254 228 282

–

Example: **Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.**

270 273 258 204 254 228 282

- **The data has no outliers, and based on a histogram, we can assume that the data are from a population with a normal distribution.**

Example: **Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.**

270 273 258 204 254 228 282

$n = 7$ df = 6

$\bar{x} = 252.7$ lb

$s = 27.6$ lb

Example: **Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.**

270 273 258 204 254 228 282

$n = 7$ df = 6

$\bar{x} = 252.7$ lb

$s = 27.6$ lb

While the data has a mean that is greater than 165 lbs., is this mean *significantly* greater than 165 or is it possibly a chance fluctuation?

Example: Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.

270 273 258 204 254 228 282

$n = 7$ df = 6

$\bar{x} = 252.7$ lb

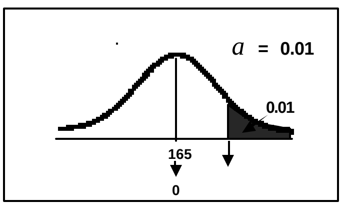
$s = 27.6$ lb

Claim: $\mu > 165$ lb

H_0: $\mu = 165$ lb

H_1: $\mu > 165$ lb

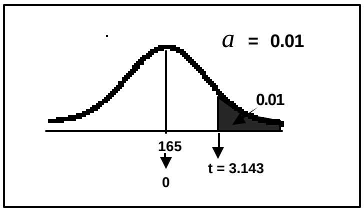
(right tailed test)

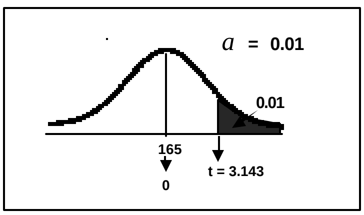


From A-3, row 6 (n -1 = 6)
column 0.01 (one tail)

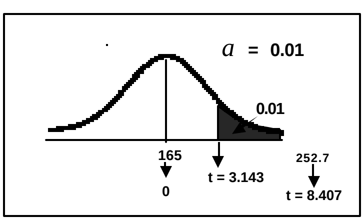
Table A-3 t Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
12	3.054	2.681	2.179	1.782	1.356	.696
13	3.012	2.650	2.160	1.771	1.350	.694
14	2.977	2.625	2.145	1.761	1.345	.692
15	2.947	2.602	2.132	1.753	1.341	.691
16	2.921	2.584	2.120	1.746	1.337	.690
17	2.898	2.567	2.110	1.740	1.333	.689
18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684
27	2.771	2.473	2.052	1.703	1.314	.684
28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large (z)	2.575	2.327	1.960	1.645	1.282	.675

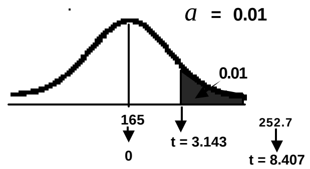




$$t = \frac{\bar{x} - \mu_{\bar{x}}}{s/\sqrt{n}} = \frac{252.7 - 165}{27.6/\sqrt{7}} = 8.407$$



$$t = \frac{\bar{x} - \mu_{\bar{x}}}{s/\sqrt{n}} = \frac{252.7 - 165}{27.6/\sqrt{7}} = 8.407$$



Reject H_o

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{s/\sqrt{n}} = \frac{252.7 - 165}{27.6/\sqrt{7}} = 8.407$$

Example: Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.

270 273 258 204 254 228 282

$n = 7$ df = 6

$\bar{x} = 252.7$ lb

$s = 27.6$ lb

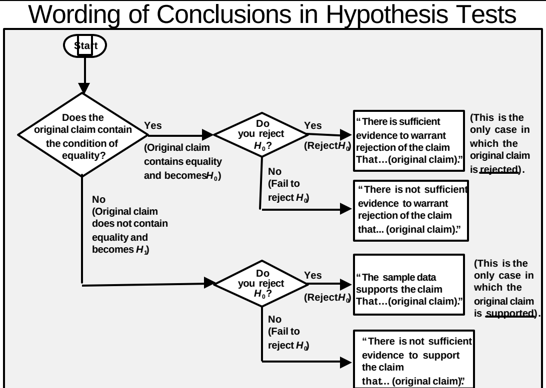
Claim: $\mu > 165$ lb

Reject H_0: $\mu = 165$ lb

H_1: $\mu > 165$ lb

(right tailed test)

Wording of Conclusions in Hypothesis Tests



Example: Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.

270 273 258 204 254 228 282

Final conclusion:
There is sufficient evidence to support the claim that the sample comes from a population with a mean greater than 165 lbs.

Claim: $\mu > 165$ lb

Reject H_0: $\mu = 165$ lb

H_1: $\mu > 165$ lb

(right tailed test)

With the Student *t*-distributions, the sample evidence must be <u>more extreme</u> to become a significant difference (especially with small samples.)

REMEMBER:

Critical t scores that are located below the assumed population mean (left-tail) will be negative.

P-Value Method

- Table A-3 includes only selected values of α
- Specific *P*-values usually cannot be found
- Use Table A - 3 to identify range of *P*-values
- Some calculators and computer programs will find exact *P*-values

Example: A former statistics student of the text author randomly selected 16 new textbooks in the college bookstore. She found these books had prices with a mean of $70.41 and a standard deviation of $19.70. Is there sufficient evidence to warrant rejection of the claim in the college catalog that the mean price of a textbook at this college is less than $75?