Testing a Claim about a Standard Deviation or Variance

Assumptions
for testing claims about a population standard deviation or variance
1) The sample is a simple random sample.
2) The population has a normal distribution (a strict requirement).

Chi-Square Distribution
Test Statistic

\[ X^2 = \frac{(n - 1) S^2}{\sigma^2} \]
Chi-Square Distribution

Test Statistic

\[ X^2 = \frac{(n - 1) s^2}{\sigma^2} \]

- \( n \) = sample size
- \( s^2 \) = sample variance
- \( \sigma^2 \) = population variance (given in null hypothesis)

Critical Values and P-values for Chi-Square Distribution

- Found in Table A-4
- Degrees of freedom = \( n - 1 \)
- Based on cumulative areas from the RIGHT

Properties of Chi-Square Distribution

- All values of \( X^2 \) are nonnegative, and the distribution is not symmetric.
- There is a different distribution for each number of degrees of freedom.
- The critical values are found in Table A-4 using \( n - 1 \) degrees of freedom.
Properties of Chi-Square Distribution

- Not symmetric
- All values are nonnegative

Figure 8-13

Chi-Square Distribution for 10 and 20 Degrees of Freedom

- df = 10
- df = 20

Figure 8-14

There is a different distribution for each number of degrees of freedom.

Table A-4: Critical values are found by determining the area to the RIGHT of the critical value.

<table>
<thead>
<tr>
<th>df</th>
<th>$\alpha/2$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.025</td>
<td>0.05</td>
</tr>
</tbody>
</table>

$\alpha/2 = 0.025$
Table A-4: Critical values are found by determining the area to the RIGHT of the critical value.

\[
\begin{align*}
\alpha &= 0.05 \\
\alpha/2 &= 0.025 \\
df &= 80 \\
0.025 &\quad 0.025 \\
106.629 &\quad 106.629
\end{align*}
\]

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\alpha &= 0.05 \\
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\[
\begin{align*}
\alpha &= 0.05 \\
\alpha/2 &= 0.025 \\
df &= 80 \\
0.025 &\quad 0.025 \\
57.153 &\quad 106.629
\end{align*}
\]
Example: Aircraft altimeters have measuring errors with a standard deviation of 43.7 ft. With new production equipment, 81 altimeters measure errors with a standard deviation of 52.3 ft. Use the 0.05 significance level to test the claim that the new altimeters have a standard deviation different from the old value of 43.7 ft.

Claim: \( \sigma = 43.7 \)

\[ H_0: \sigma = 43.7 \]
\[ H_1: \sigma \neq 43.7 \]

\( \alpha = 0.05 \)
\[ \alpha/2 = 0.025 \]

\( n = 81 \)
\( df = 80 \)

Table A-4
\[ x^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(81 - 1)(52.3)^2}{43.7^2} = 114.586 \]

**Example:** Aircraft altimeters have measuring errors with a standard deviation of 43.7 ft. With new production equipment, 81 altimeters measure errors with a standard deviation of 52.3 ft. Use the 0.05 significance level to test the claim that the new altimeters have a standard deviation different from the old value of 43.7 ft.

Claim: \( \sigma \neq 43.7 \)

REJECT \( H_0: \sigma = 43.7 \)

\( H_1: \sigma \neq 43.7 \)
Example: Aircraft altimeters have measuring errors with a standard deviation of 43.7 ft. With new production equipment, 81 altimeters measure errors with a standard deviation of 52.3 ft. Use the 0.05 significance level to test the claim that the new altimeters have a standard deviation different from the old value of 43.7 ft.

SUPPORT: Claim: \( \sigma \neq 43.7 \)
REJECT: \( H_0: \sigma = 43.7 \)
\( H_1: \sigma \neq 43.7 \)

The sample evidence supports the claim that the standard deviation is different from 43.7 ft.

The new production method appears to be worse than the old method. The data supports that there is more variation in the error readings than before.
P-Value Method

- Table A-4 includes only selected values of \( \alpha \)
- Specific \( P \)-values usually cannot be found
- Use Table A-4 to identify limits that contain the \( P \)-value
- Some calculators and computer programs will find exact \( P \)-values

Example: Tests in the author’s past statistics classes have scores with a standard deviation equal to 14.1. One of his recent classes has 27 test scores with a standard deviation of 9.3. Use the 0.01 significance level to test the claim that this current class has less variation than past classes. Does a lower standard deviation suggest that the current class is doing better?