8-6

Testing a Claim about a Standard Deviation or Variance

Assumptions

for testing claims about a population standard deviation or variance

1) The sample is a simple random sample.

2) The population has a normal distribution (a strict requirement).

Chi-Square Distribution

Test Statistic

$$X^2 = \frac{(n-1)s^2}{\sigma^2}$$

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Test Statistic

$$X^2 = \frac{(n-1)\,s^2}{\sigma^2}$$

n = sample size

s^2 = sample variance

σ^2 = population variance (given in null hypothesis)

Critical Values and P-values for Chi-Square Distribution

- ❖ Found in Table A-4
- ❖ Degrees of freedom = n -1
- ❖ Based on cumulative areas from the RIGHT

Properties of Chi-Square Distribution

- ❖ All values of X^2 are nonnegative, and the distribution is not symmetric.
- ❖ There is a different distribution for each number of degrees of freedom.
- ❖ The critical values are found in Table A-4 using n-1 degrees of freedom.

Properties of Chi-Square Distribution

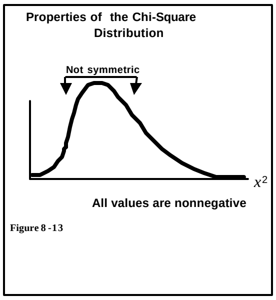


Figure 8-13

Properties of Chi-Square Distribution

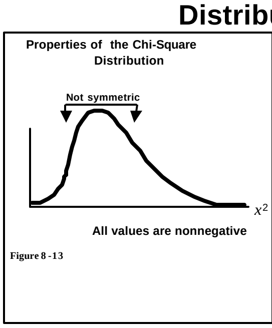


Figure 8-13

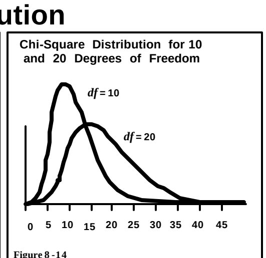
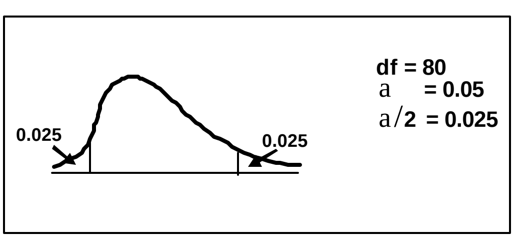


Figure 8-14

There is a different distribution for each number of degrees of freedom.

Table A-4: Critical values are found by determining the area to the RIGHT of the critical value.



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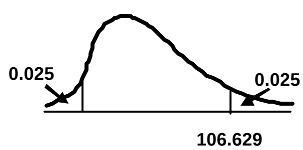

df = 80
a **= 0.05**
a/**2 = 0.025**

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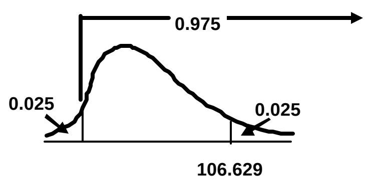

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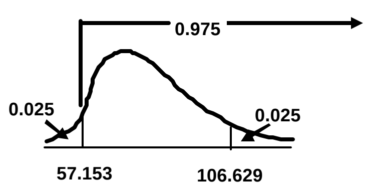

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Example: **Aircraft altimeters have measuring errors with a standard deviation of 43.7 ft. With new production equipment, 81 altimeters measure errors with a standard deviation of 52.3 ft. Use the 0.05 significance level to test the claim that the new altimeters have a standard deviation different from the old value of 43.7 ft.**

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Claim: s ¹ **43.7**

H_0: $s = 43.7$ $\quad$ a **= 0.05** $\quad$ a/**2** **= 0.025**

H_1: s ¹ **43.7**

n **= 81**

df **= 80**

Table A-4

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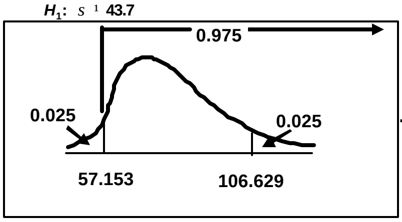

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Table A-4

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(81-1)\,(52.3)^2}{43.7^2} \approx 114.586$$

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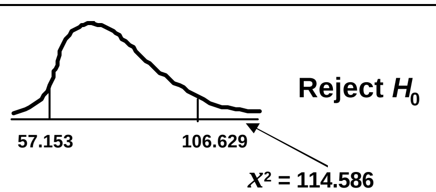

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Claim: $\sigma \neq 43.7$

REJECT H_0: $\sigma = 43.7$

H_1: $\sigma \neq 43.7$

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SUPPORT **Claim:** $s \;^{1}$ **43.7**

REJECT H_0: s = **43.7**

H_1: $s \;^{1}$ **43.7**

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SUPPORT **Claim:** $s \;^{1}$ **43.7**

REJECT H_0: s = **43.7**

H_1: $s \;^{1}$ **43.7**

The sample evidence supports the claim that the standard deviation is different from 43.7 ft.

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SUPPORT **Claim:** $s \;^{1}$ **43.7**

REJECT H_0: s = **43.7**

H_1: $s \;^{1}$ **43.7**

The new production method appears to be worse than the old method. The data supports that there is more variation in the error readings than before.

P-Value Method

- Table A-4 includes only selected values of α
- Specific *P*-values usually cannot be found
- Use Table A- 4 to identify <u>limits</u> that contain the *P*-value
- Some calculators and computer programs will find exact *P*-values

Example: Tests in the author's past statistics classes have scores with a standard deviation equal to 14.1 One of his recent classes has 27 test scores with a standard deviation of 9.3. Use the 0.01 significance level to test the claim that this current class has less variation than past classes. Does a lower standard deviation suggest that the current class is doing better?