6 - 4

Estimating a Population Mean: σ Not Known

Assumptions

σ Not Known

1) The sample is a simple random sample.
2) Either the sample is from a normally distributed population or $n > 30$.

1

The sample mean $\overline{x}$ is the best point estimate of the population mean μ.

Use the Student t Distribution

Student *t* Distribution

If the distribution of a population is essentially normal, then the distribution of

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Central Limit Theorem

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

Student *t* Distribution

If the distribution of a population is essentially normal, then the distribution of

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

- **is essentially a <u>Student *t* Distribution</u> for all samples of size *n*.**
- **is used to find critical values denoted by** $t_{\alpha/2}$

Table A - 3

- Formulas and Tables Card
- Appendix

Table A-3 *t* Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
12	3.054	2.681	2.179	1.782	1.356	.696
13	3.012	2.650	2.160	1.771	1.350	.694
14	2.977	2.625	2.145	1.761	1.345	.692
15	2.947	2.602	2.132	1.753	1.341	.691
16	2.921	2.584	2.120	1.746	1.337	.690
17	2.898	2.567	2.110	1.740	1.333	.689
18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684
27	2.771	2.473	2.052	1.703	1.314	.684
28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large (z)	2.575	2.327	1.960	1.645	1.282	.675

NEGATIVE Z Scores Table A-2

TABLE A-2 Standard Normal (*z*) Distribution: Cumulative Area from the LEFT

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.50 and lower	.0001									
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051 *	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505 *	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559

Definition

Degrees of Freedom (*df*)

corresponds to the number of sample values that can vary after certain restrictions have imposed on all data values

$$df = n - 1$$

with this procedure

Definition

Degrees of Freedom (*df*) = *n* - 1

corresponds to the number of sample values that can vary after certain restrictions have imposed on all data values

Any #	Any #	Any #	Any #	Any #	Any #	Any #	Any #	Any #	Specific #

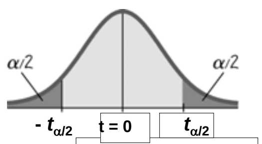
$n = 10$ ➡ $df = 10 - 1 = 9$

so that $\bar{x} = 80$

Example

Find the critical value $t_{\alpha/2}$ for sample size n = 15 corresponding to a 95% confidence level

Finding the Critical Value $t_{\alpha/2}$



n = 15

df = 14

Degree of confidence = 95%

α = 5%

$\alpha/2$ = 0.025

Table A-3 *t* Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
12	3.054	2.681	2.179	1.782	1.356	.696
13	3.012	2.650	2.160	1.771	1.350	.694
14	2.977	2.625	2.145	1.761	1.345	.692
15	2.947	2.602	2.132	1.753	1.341	.691
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18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684
27	2.771	2.473	2.052	1.703	1.314	.684
28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large (z)	2.575	2.327	1.960	1.645	1.282	.675

Example

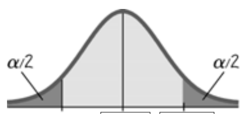
Find the critical value $t_{\alpha/2}$ for sample size n = 15 corresponding to a 95% confidence level

df = 14

Area of 0.05 in two tails

$t_{\alpha/2}$ = 2.145

The Critical Value $t_{\alpha/2}$



Margin of Error E for Estimate of μ

Based on an Unknown σ and a Small Simple Random Sample from a Normally Distributed Population

Formula 6-6

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}}$$

where $t_{\alpha/2}$ has n - 1 degrees of freedom

Confidence Interval for the Estimate of μ

Based on an Unknown σ and a Small Simple Random Sample from a Normally Distributed Population

$$\bar{x} - E < \mu < \bar{x} + E$$

where $E = t_{\alpha/2} \frac{s}{\sqrt{n}}$

$t_{\alpha/2}$ found in Table A-3

Procedure for Constructing a Confidence Interval for μ when σ is not known

1. Verify that the required assumptions are met.

2. Using n - 1 degrees of freedom, refer to Table A-3 and find the critical value $t_{\alpha/2}$ that corresponds to the desired degree of confidence.

3. Evaluate the margin of error $E = t_{\alpha/2} \cdot s / \sqrt{n}$.

4. Find the values of $\bar{x}$ - E and $\bar{x}$ + E. Substitute those values in the general format for the confidence interval:

$$\bar{x} - E < \mu < \bar{x} + E$$

5. Round the resulting confidence interval limits.

Important Properties of the Student *t* Distribution

1. The Student *t* distribution is different for different sample sizes (see Figure 6-5 for the cases n = 3 and n = 12).
2. The Student *t* distribution has the same general symmetric bell shape as the normal distribution but it reflects the greater variability (with wider distributions) that is expected with small samples.
3. The Student *t* distribution has a mean of t = 0 (just as the standard normal distribution has a mean of z = 0).
4. The standard deviation of the Student *t* distribution varies with the sample size and is greater than 1 (unlike the standard normal distribution, which has a σ = 1).
5. As the sample size n gets larger, the Student *t* distribution gets closer to the normal distribution.

Student *t* Distributions for n = 3 and n = 12

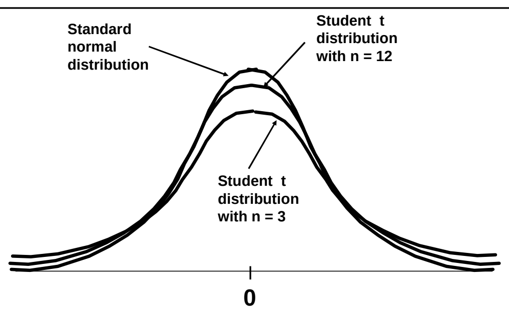


Figure 6-5

Example: **A study of 12 Dodge Vipers involved in collisions resulted in repairs averaging $26,227 and a standard deviation of $15,873. Find the 95% interval estimate of μ, the mean repair cost for all Dodge Vipers involved in collisions. (The 12 cars' distribution appears to be bell-shaped.)**

$\bar{x} = 26{,}227$

$s = 15{,}873$

$\alpha = 0.05$

$\alpha/2 = 0.025$

Table A-3 *t* Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
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18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
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28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large (z)	2.575	2.327	1.960	1.645	1.282	.675

8

Example: **A study of 12 Dodge Vipers involved in collisions resulted in repairs averaging $26,227 and a standard deviation of $15,873. Find the 95% interval estimate of μ, the mean repair cost for all Dodge Vipers involved in collisions. (The 12 cars' distribution appears to be bell-shaped.)**

$\bar{x} = 26{,}227$

$s = 15{,}873$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$t_{\alpha/2} = 2.201$

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}} = \frac{(2.201)(15{,}873)}{\sqrt{12}} = 10{,}085.3$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$26{,}227 - 10{,}085.3 < \mu < 26{,}227 + 10{,}085.3$$

$$\$16{,}141.7 < \mu < \$36{,}312.3$$

We are 95% confident that this interval contains the average cost of repairing a Dodge Viper.