Chapter 6 and 7

Review for Exam

Chapter 6

Estimates and Sample Sizes

Definition

Confidence Interval

(or Interval Estimate)

a range (or an interval) of values <u>used to estimate</u> the true value of the population parameter

Lower # < population parameter < Upper #

As an example

$$0.476 < p < 0.544$$

Confidence Interval for Population Proportion

$$\hat{p} - E < p < \hat{p} + E$$

where

$$E = z_{\alpha/2}\sqrt{\frac{\hat{p}\,\hat{q}}{n}}$$

Notation for Proportions

$p =$ population proportion

$\hat{p} = \frac{x}{n}$ sample proportion of x successes in a sample of size n

(pronounced 'p-hat')

$\hat{q} = 1 - \hat{p} =$ sample proportion of x failures in a sample size of n

Round-Off Rule for Confidence Interval Estimates of *p*

Round the confidence interval limits to

three significant digits

Procedure for Constructing a Confidence Interval for *p*

1. **Verify that the required assumptions are satisfied. (The sample is a simple random sample, the conditions for the binomial distribution are satisfied, and the normal distribution can be used to approximate the distribution of sample proportions because $np \geq 5$, and $nq \geq 5$ are both satisfied).**
2. **Refer to Table A-2 and find the critical value $z_{\alpha/2}$ that corresponds to the desired confidence level.**
3. **Evaluate the margin of error $E = \sqrt{\frac{\hat{p}\hat{q}}{n}}$**

Procedure for Constructing a Confidence Interval for *p*

4. **Using the calculated margin of error, E and the value of the sample proportion, $\hat{p}$, find the values of $\hat{p} - E$ and $\hat{p} + E$. Substitute those values in the general format for the confidence interval:**

$$\hat{p} - E < p < \hat{p} + E$$

5. **Round the resulting confidence interval limits to three significant digits.**

Example: **In the Chapter Problem, we noted that 829 adult Minnesotans were surveyed, and 51% of them are opposed to the use of the photo-cop for issuing traffic tickets. Use these survey results.**

Find the 95% confidence interval estimate of the population proportion *p*.

Example: In the Chapter Problem, we noted that 829 adult Minnesotans were surveyed, and 51% of them are opposed to the use of the photo-cop for issuing traffic tickets. Use these survey results.

First, we check for assumptions. We note that $n\hat{p} = 422.79 \geq 5$, and $n\hat{q} = 406.21 \geq 5$.

Next, we calculate the margin of error. We have found that $\hat{p} = 0.51$, $\hat{q} = 1 - 0.51 = 0.49$, $z_{\alpha/2} = 1.96$, and $n = 829$.

$$E = 1.96\sqrt{\frac{(0.51)(0.49)}{829}}$$

$$E = 0.03403$$

Example: In the Chapter Problem, we noted that 829 adult Minnesotans were surveyed, and 51% of them are opposed to the use of the photo-cop for issuing traffic tickets. Use these survey results.

Find the 95% confidence interval for the population proportion *p*.

We substitute our values from Part a to obtain:

$$0.51 - 0.03403 < p < 0.51 + 0.03403,$$
$$0.476 < p < 0.544$$

Example: In the Chapter Problem, we noted that 829 adult Minnesotans were surveyed, and 51% of them are opposed to the use of the photo-cop for issuing traffic tickets. Use these survey results.

Based on the results, can we safely conclude that the majority of adult Minnesotans oppose use of the photo-cop?

Based on the survey results, we are 95% confident that the limits of 47.6% and 54.4% contain the true percentage of adult Minnesotans opposed to the photo-cop. The percentage of opposed adult Minnesotans is likely to be any value between 47.6% and 54.4%. However, a majority requires a percentage greater than 50%, so we *cannot* safely conclude that the majority is opposed (because the *entire* confidence interval is not greater than 50%).

Estimating a Population Mean: σ Not Known

Confidence Interval for the Estimate of μ

Based on an Unknown σ and a Small Simple Random Sample from a Normally Distributed Population

$$\bar{x} - E < \mu < \bar{x} + E$$

where $E = t_{\alpha/2} \frac{s}{\sqrt{n}}$

$t_{\alpha/2}$ found in Table A-3

Table A-3 *t* Distribution

Degrees of freedom	.005 (one tail) .01 (two tails)	.01 (one tail) .02 (two tails)	.025 (one tail) .05 (two tails)	.05 (one tail) .10 (two tails)	.10 (one tail) .20 (two tails)	.25 (one tail) .50 (two tails)
1	63.657	31.821	12.706	6.314	3.078	1.000
2	9.925	6.965	4.303	2.920	1.886	.816
3	5.841	4.541	3.182	2.353	1.638	.765
4	4.604	3.747	2.776	2.132	1.533	.741
5	4.032	3.365	2.571	2.015	1.476	.727
6	3.707	3.143	2.447	1.943	1.440	.718
7	3.500	2.998	2.365	1.895	1.415	.711
8	3.355	2.896	2.306	1.860	1.397	.706
9	3.250	2.821	2.262	1.833	1.383	.703
10	3.169	2.764	2.228	1.812	1.372	.700
11	3.106	2.718	2.201	1.796	1.363	.697
12	3.054	2.681	2.179	1.782	1.356	.696
13	3.012	2.650	2.160	1.771	1.350	.694
14	2.977	2.625	2.145	1.761	1.345	.692
15	2.947	2.602	2.132	1.753	1.341	.691
16	2.921	2.584	2.120	1.746	1.337	.690
17	2.898	2.567	2.110	1.740	1.333	.689
18	2.878	2.552	2.101	1.734	1.330	.688
19	2.861	2.540	2.093	1.729	1.328	.688
20	2.845	2.528	2.086	1.725	1.325	.687
21	2.831	2.518	2.080	1.721	1.323	.686
22	2.819	2.508	2.074	1.717	1.321	.686
23	2.807	2.500	2.069	1.714	1.320	.685
24	2.797	2.492	2.064	1.711	1.318	.685
25	2.787	2.485	2.060	1.708	1.316	.684
26	2.779	2.479	2.056	1.706	1.315	.684
27	2.771	2.473	2.052	1.703	1.314	.684
28	2.763	2.467	2.048	1.701	1.313	.683
29	2.756	2.462	2.045	1.699	1.311	.683
Large (z)	2.575	2.327	1.960	1.645	1.282	.675

Example: A study of 12 Dodge Vipers involved in collisions resulted in repairs averaging $26,227 and a standard deviation of $15,873. Find the 95% interval estimate of μ, the mean repair cost for all Dodge Vipers involved in collisions. (The 12 cars' distribution appears to be bell-shaped.)

$\bar{x} = 26{,}227$

$s = 15{,}873$

$\alpha = 0.05$

$\alpha/2 = 0.025$

$t_{\alpha/2} = 2.201$

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}} = \frac{(2.201)(15{,}873)}{\sqrt{12}} = 10{,}085.3$$

$$\bar{x} - E < \mu < \bar{x} + E$$

$$26{,}227 - 10{,}085.3 < \mu < 26{,}227 + 10{,}085.3$$

$$\$16{,}141.7 < \mu < \$36{,}312.3$$

We are 95% confident that this interval contains the average cost of repairing a Dodge Viper.

End of 6-2 and 6-3

Determining Sample Size Required to Estimate

p and μ

Sample Size for Estimating Proportion *p*

When an estimate of $\hat{p}$ is known:

$$n = \frac{(z_{\alpha/2})^2 \hat{p}\hat{q}}{E^2}$$ Formula 6-2

When no estimate of $\hat{p}$ is known:

$$n = \frac{(z_{\alpha/2})^2 \, 0.25}{E^2}$$ Formula 6-3

Example: We want to determine, with a margin of error of four percentage points, the current percentage of U.S. households using e-mail. Assuming that we want 90% confidence in our results, how many households must we survey? A 1997 study indicates 16.9% of U.S. households used e-mail.

$$n = \frac{[z_{\alpha/2}]^2 \hat{p}\hat{q}}{E^2}$$

$$= \frac{[1.645]^2 (0.169)(0.831)}{0.04^2}$$

$= 237.51965$

$= 238$ households

To be 90% confident that our sample percentage is within four percentage points of the true percentage for all households, we should randomly select and survey 238 households.

Example: We want to determine, with a margin of error of four percentage points, the current percentage of U.S. households using e-mail. Assuming that we want 90% confidence in our results, how many households must we survey? There is no prior information suggesting a possible value for the sample percentage.

$$n = \frac{[z_{\alpha/2}]^2 (0.25)}{E^2}$$

$$= \frac{(1.645)^2 (0.25)}{0.04^2}$$

$= 422.81641$

$= 423$ households

With no prior information, we need a larger sample to achieve the same results with 90% confidence and an error of no more than 4%.

Sample Size for Estimating Mean μ

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

(solve for n by algebra)

$$n = \left[\frac{z_{\alpha/2}\ \sigma}{E}\right]^2$$

Formula 6-5

$z_{\alpha/2}$ = critical z score based on the desired degree of confidence

E = desired margin of error

σ = population standard deviation

Example: If we want to estimate the mean weight of plastic discarded by households in one week, how many households must be randomly selected to be 99% confident that the sample mean is within 0.25 lb of the true population mean? (A previous study indicates the standard deviation is 1.065 lb.)

$\alpha = 0.01$
$z_{\alpha/2} = 2.575$
$E = 0.25$
$s = 1.065$

$$n = \left[\frac{z_{\alpha/2}\,\sigma}{E}\right]^2 = \left[\frac{(2.575)(1.065)}{0.25}\right]^2$$

$$= 120.3 = 121 \text{ households}$$

We would need to randomly select 121 households and obtain the average weight of plastic discarded in one week. We would be 99% confident that this mean is within 1/4 lb of the population mean.

Chapter 7

Hypothesis Testing

Claim: Using math symbols

H_0: Must contain equality

H_1: Will contain $\neq$, <, >

Test Statistic

The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

Test statistic for proportions

Test Statistic

The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{\frac{s}{\sqrt{n}}}$$

Test statistic for mean

Test Statistic

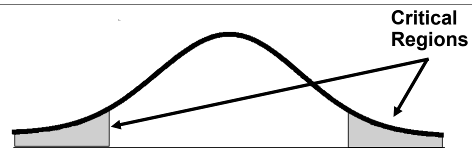
The test statistic is a value computed from the sample data, and it is used in making the decision about the rejection of the null hypothesis.

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Test statistic for standard deviation

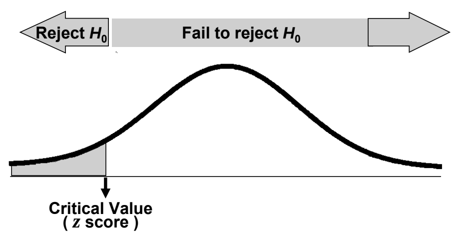
Critical Region

Set of all values of the test statistic that would cause a rejection of the null hypothesis



Critical Value

Any value that separates the critical region (where we reject the null hypothesis) from the values of the test statistic that do not lead to a rejection of the null hypothesis



Two-tailed, Right-tailed, Left-tailed Tests

The tails in a distribution are the extreme regions bounded by critical values.

Decision Criterion

Traditional method:

<u>*Reject H_0*</u> if the test statistic falls within the critical region.

<u>*Fail to reject H_0*</u> if the test statistic does not fall within the critical region.

Wording of Final Conclusion

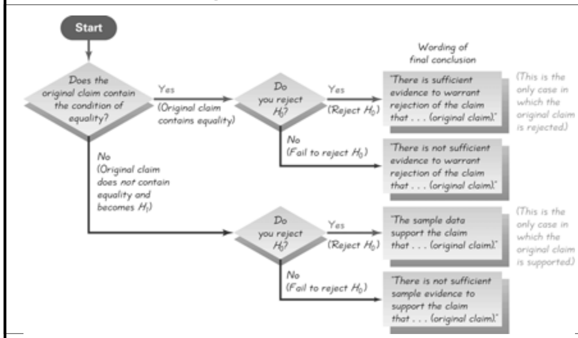
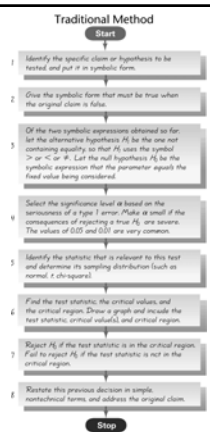


Figure 7-7



Comprehensive Hypothesis Test

Example: It was found that 821 crashes of midsize cars equipped with air bags, 46 of the crashes resulted in hospitalization of the drivers. Using the 0.01 significance level, test the claim that the air bag hospitalization is lower than the 7.8% rate for cars with automatic safety belts.

Claim: $p < 0.078$

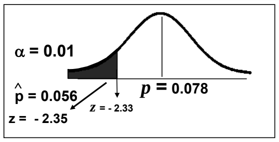
H_0: $p = 0.078$

H_1: $p < 0.078$

reject H_0

$\hat{p} = 46 / 821 = 0.0560$

$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{0.056 - 0.078}{\sqrt{\frac{(0.078)(0.922)}{821}}} \approx -2.35$$



There is sufficient evidence to support claim that the air bag hospitalization rate is lower than the 7.8% rate for automatic safety belts.

7-5

Testing a Claim about a Mean: σ Not Known

Example: Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.

270 273 258 204 254 228 282

$n = 7$ df = 6

$\bar{x} = 252.7$ lb

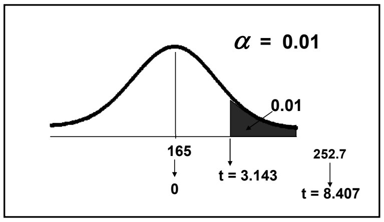
$s = 27.6$ lb

Claim: $\mu > 165$ lb

H_0: $\mu = 165$ lb

H_1: $\mu > 165$ lb

(right tailed test)



Reject H_o

$$t = \frac{\bar{x} - \mu_{\bar{x}}}{s/\sqrt{n}} = \frac{252.7 - 165}{27.6/\sqrt{7}} = 8.407$$

Example: Seven axial load scores are listed below. At the 0.01 level of significance, test the claim that this sample comes from a population with a mean that is greater than 165 lbs.

270 273 258 204 254 228 282

Final conclusion:
There is sufficient evidence to support the claim that the sample comes from a population with a mean greater than 165 lbs.

Claim: $\mu > 165$ lb

Reject H_0: $\mu = 165$ lb

H_1: $\mu > 165$ lb

(right tailed test)

7-6

Testing a Claim about a Standard Deviation or Variance

Chi-Square Distribution

Test Statistic

$$X^2 = \frac{(n - 1)\, s^2}{\sigma^2}$$

n = sample size

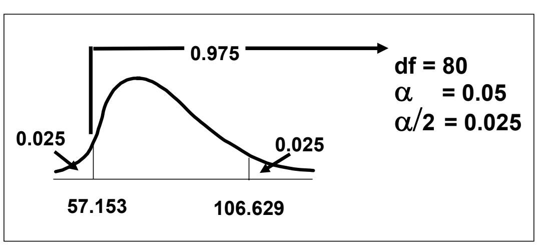
s^2 = sample variance

σ^2 = population variance (given in null hypothesis)

Critical Values and P-values for Chi-Square Distribution

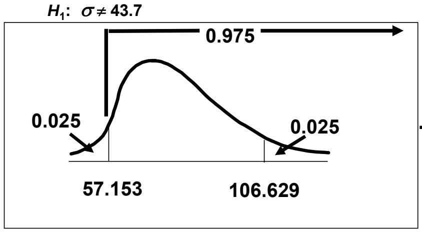
- ❖ Found in Table A-4
- ❖ Degrees of freedom = n -1
- ❖ Based on cumulative areas from the RIGHT

Table A-4: Critical values are found by determining the area to the RIGHT of the critical value.



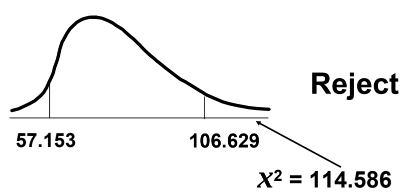
Example: Aircraft altimeters have measuring errors with a standard deviation of 43.7 ft. With new production equipment, 81 altimeters measure errors with a standard deviation of 52.3 ft. Use the 0.05 significance level to test the claim that the new altimeters have a standard deviation different from the old value of 43.7 ft.

Claim: $\sigma \neq 43.7$
H_0: $\sigma = 43.7$ $\alpha = 0.05$ $\alpha/2 = 0.025$
H_1: $\sigma \neq 43.7$



$n = 81$
$df = 80$
Table A-4

$$X^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(81-1)(52.3)^2}{43.7^2} \approx 114.586$$



Reject H_0

Example: Aircraft altimeters have measuring errors with a standard deviation of 43.7 ft. With new production equipment, 81 altimeters measure errors with a standard deviation of 52.3 ft. Use the 0.05 significance level to test the claim that the new altimeters have a standard deviation different from the old value of 43.7 ft.

SUPPORT **Claim: $\sigma \neq 43.7$**
REJECT **H_0: $\sigma = 43.7$**
H_1: $\sigma \neq 43.7$

The new production method appears to be worse than the old method. The data supports that there is more variation in the error readings than before.

Table 7-3	Hypothesis Tests		
Parameter	**Conditions**	**Distribution and Test Statistic**	**Critical and P-values**
Proportion	**np ≥ 5 and nq ≥ 5**	**Normal:** $z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$	**Table A-2**
Mean	**σ not known and normally distributed or n ≥ 30**	**Student t:** $t = \frac{\bar{x} - \mu_x}{\frac{s}{\sqrt{n}}}$	**Table A-3**
Standard Deviation or Variance	**Population normally distributed**	**Chi-Square:** $\chi^2 = \frac{(n-1)s^2}{\sigma^2}$	**Table A-4**