

Gauss-Jordan Elimination

- I. A linear system such as
$$\begin{cases} x + 5y - z = -11 \\ 3z = 12 \\ 2x + 4y - 2z = 8 \end{cases}$$
 can be solved by hand algebraically or by using an augmented matrix and elementary row operations.

A. Elementary row operations that produce row-equivalent matrices

1. Two rows are interchanged $R_i \leftrightarrow R_j$
 2. A row is multiplied by a nonzero constant $kR_i \rightarrow R_i$
 3. A constant multiple of one row is added to another row $kR_j + R_i \rightarrow R_i$
- (NOTE : \rightarrow means "replaces")

II. Performing elementary row operations on the TI-83

The result of a row operation is displayed on the home screen, but it is not automatically stored! You should immediately store the result under a different name. It is convenient (and frequently useful) to store the results alphabetically.

A. Row swap

To interchange rows 1 and 3 of matrix A:

MATRIX **MATH** C:rowSwap(**ENTER** **MATRIX**
 NAMES 1:[A] **ENTER** **1** **ENTER** **3** **ENTER**
ENTER **STO\$** **MATRIX** NAMES 2:[B] **ENTER**

```
rowSwap([A],1,3)
[[1 5 -1 -11]
 [2 4 -2 8 ]
 [0 0 3 12 ]]
```

B. Multiplying a row by a nonzero scalar

To multiply row 1 of matrix A by $\frac{1}{3}$:

MATRIX **MATH** E:*row(**ENTER** $\frac{1}{3}$ **ENTER** **MATRIX**
 NAMES 1:[A] **ENTER** **1** **ENTER** **ENTER**
STO\$ **MATRIX** NAMES 3:[C] **ENTER**

```
*row(1/3,[A],1)
[[0 0 1 4 ]
 [2 4 -2 8 ]
 [1 5 -1 -11]]
```

C. Adding a nonzero scalar multiple of one row to another row

To multiply row 2 of matrix A by $-\frac{1}{2}$ and add it to row 3
 (of matrix A):

MATRIX **MATH** F:*row+(**ENTER** $-\frac{1}{2}$ **ENTER** **MATRIX**
 NAMES 1:[A] **ENTER** **2** **ENTER** **3** **ENTER** **ENTER**
STO\$ **MATRIX** NAMES 4:[D] **ENTER**

```
*row+(-1/2,[A],2,3)
[[0 0 3 12 ]
 [2 4 -2 8 ]
 [0 3 0 -15]]
```

III. Solve the system of equations represented by the given augmented matrix using the given row operations:

As you perform each row operation, record the result, and store it as indicated.

$$\text{Let } [A] = \left[\begin{array}{ccc|c} 0 & 0 & 3 & 12 \\ 2 & 4 & -2 & 8 \\ 1 & 5 & -1 & -11 \end{array} \right]$$

	matrix operation	result	store as matrix
A.	$R_1 \leftrightarrow R_3$		[B]
B.	$-2R_1 + R_2 \rightarrow R_2$		[C]
C.	$-\frac{1}{6}R_2 \rightarrow R_2$		[D]
D.	$-5R_2 + R_1 \rightarrow R_1$		[E]
E.	$\frac{1}{3}R_3 \rightarrow R_3$		[F]
F.	$R_3 + R_1 \rightarrow R_1$		[G]

The solution (x, y, z) should be in the column to the right of the bar. $(x, y, z) = \underline{\hspace{2cm}}$

IV. This system could have been solved using different row operations and/or the same row operations in other orders. To minimize the amount of work necessary to solve the system, you must be careful not to backtrack and “undo” work which you have already done. The process we have been

using is ideal for use with calculator and/or computer programs. Therefore, it is imperative that we develop an algorithm that will **always** work. The most commonly used such algorithm is the **Gauss-Jordan Elimination Method**.

- A. Get a "1" in position 1,1. Then use row 1 to get "0"s in the rest of column 1.
- B. Get a "1" in position 2,2. Then use row 2 to get "0"s in the rest of column 2.
- C. Get a "1" in position 3,3. Then use row 3 to get "0"s in the rest of column 3.

V. You will be required to solve one problem on the test by showing the individual row operations as demonstrated above. You may use your calculators to do the actual calculations on each step.

VI. Study your owners manual to see if your calculator has a "row reduced form" command. If so, it will save you a lot of time and effort! If not, practice until you can do the necessary steps quickly and accurately. On the TI-83, matrix A can be changed from augmented form to row reduced form using: `(MATRIX) MATH B:rref((ENTER) (MATRIX) NAMES 1:[A] (ENTER)`

VII. Not all linear systems have solutions

A. Solve:
$$\begin{cases} 2x_1 + 6x_2 = -3 \\ x_1 + 3x_2 = 2 \end{cases}$$

B. The augmented matrix is $\left[\begin{array}{cc|c} 2 & 6 & -3 \\ 1 & 3 & 2 \end{array} \right]$, and the row reduced form is $\left[\begin{array}{cc|c} 1 & 3 & 2 \\ 0 & 0 & -7 \end{array} \right]$

C. The last row of the row reduced form says that $0x_1 + 0x_2 = -7$, i.e. $0 = -7$. This system has no solution.

VIII. Some linear systems have multiple solutions

A. Solve
$$\begin{cases} 2x_1 - x_2 = 4 \\ -6x_1 + 3x_2 = -12 \end{cases}$$

B. The augmented matrix is $\left[\begin{array}{cc|c} 2 & -1 & 4 \\ -6 & 3 & -12 \end{array} \right]$, and the row reduced form is $\left[\begin{array}{cc|c} 1 & -1/2 & 2 \\ 0 & 0 & 0 \end{array} \right]$

C. The last row of the row reduced form says that $0x_1 + 0x_2 = 0$, which is true regardless of the values of the variables.

D. Since the bottom row is always true, we must determine when the first row is true.

1. Introduce a parameter t, and let $x_2 = t$. Then $x_1 - \frac{1}{2}x_2 = 2 \Rightarrow x_1 = \frac{1}{2}t + 2$

2. The solution to this system is $\left\{ \left(\frac{1}{2}t + 2, t \right) \mid t \in \mathbb{R} \right\}$