

## USING LINEAR PROGRAMMING TO SOLVE SYSTEMS OF EQUATIONS AND INEQUALITIES

- I. A two-dimensional linear programming problem consists of a linear **objective function** and a system of linear inequalities in two unknowns called **constraints**
- II. When all the constraints are graphed, the overlapping region is called the set of feasible solutions or the **feasible region**
- III. If a linear programming problem has a solution, it must occur at a vertex (**corner point**) of the feasible region. (The same solution may also occur at another location.)
- IV. Graphical method of solving a linear programming problem in two unknowns:
  - A. Organize given information in a chart
  - B. Write a system of constraints (inequalities) and the objective function (equation)
  - C. Graph the feasible region
  - D. Find the corner points
  - E. Substitute each corner point into the objective function to find the maximum/minimum of the function

### V. Example:

The liquid portion of a diet is to provide at least 300 calories, 36 units of vitamin A, and 90 units of vitamin C daily. A cup of dietary drink X provides 60 calories, 12 units of vitamin A, and 10 units of vitamin C. A cup of dietary drink Y provides 60 calories, 6 units of vitamin A, and 30 units of vitamin C. Now, suppose dietary drink X costs \$0.12 per cup and drink Y costs \$0.15 per cup. How many cups of each drink should be consumed each day to minimize the cost and still meet the state daily requirements?

#### A. Organize given information in a chart

|                  | # of cups per day | calories per cup | vitamin A units/cup | vitamin C units/cup | cost per cup |
|------------------|-------------------|------------------|---------------------|---------------------|--------------|
| Drink X          | x                 | 60               | 12                  | 10                  | .12          |
| Drink Y          | y                 | 60               | 6                   | 30                  | .15          |
| Minimum required |                   | 300              | 36                  | 90                  | z            |

#### B. Write constraints [inequalities]:

$$60x + 60y \geq 300$$

$$12x + 6y \geq 36$$

$$10x + 30y \geq 90$$

$$x \geq 0$$

$$y \geq 0$$

[last two constraints are **implied or physical** constraints implied by the problem]

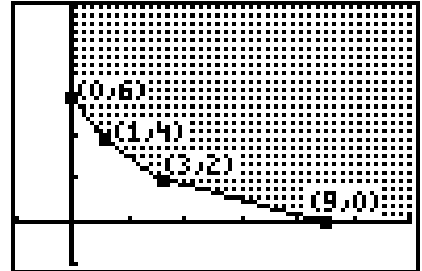
#### C. Write objective function: Minimize cost: $z = .12x + .15y$

D. Graph all constraints to find feasible region; note that the last two constraints limit the region to the first quadrant:

1. Line 1:  $60x + 60y = 300$  Intercepts: (0,5) and (5,0)
2. Line 2:  $12x + 6y = 36$  Intercepts: (0,6) and (3,0)
3. Line 3:  $10x + 30y = 90$  Intercepts: (0,3) and (9,0)

E. Find all corner points:

1. A is intersection of line 2 with y-axis (0,6)
2. B is intersection of lines 1 and 2 (1,4)
3. C is intersection of lines 1 and 3 (3,2)
4. D is intersection of line 3 with x-axis (9,0)



F. Substitute each corner point into objective function to find minimum cost:

1. At (0,6)  $z = .12(0) + .15(6) = .90$
2. At (1,4)  $z = .12(1) + .15(4) = .72$
3. At (3,2)  $z = .12(3) + .15(2) = .66$
4. At (9,0)  $z = .12(9) + .15(0) = 1.08$

G. Answer the question:

The minimum cost is \$0.66 per day, which occurs when three cups of drink X and two cups of drink Y are consumed each day.

VI. The previous example has an **unbounded** feasible region. While it has a minimum, it does not have a maximum.

VII. If a problem has a **bounded** feasible region, it will have both a maximum and a minimum. Although it is possible for the max/min to occur along a whole side of the feasible region, it is always possible to find the max/min by checking the corner points only.

## PROJECT

Follow the example in Paragraphs V and VI above to solve the following problem. Use graph paper, pencil, straightedge, etc. to make a neat accurate graph. All work must be shown for all parts of the problem and answers must be exact.

An animal feed to be mixed from soybean meal and oats must contain at least 120 lb of protein, 24 lb of fat, and 10 lb of mineral ash. Each 100-lb sack of soybean meal costs \$15 and contains 50 lb of protein, 8 lb of fat, and 5 lb of mineral ash. Each 100-lb sack of oats costs \$5 and contains 15 lb of protein, 5 lb of fat, and 1 lb of mineral ash. How many sacks of each should be used to satisfy the minimum requirements at minimum cost? What is the minimum cost?