COMPOSITE FUNCTIONS

A. Let $f(x) = \dfrac{2}{x}$ and $g(x) = \dfrac{x}{x+2}$

$D_f = (-\infty,\ 0) \cup (0,\ \infty)$ and $D_g = (-\infty,\ -2) \cup (-2,\ \infty)$

B. Find $(f \circ g)(x) = f[g(x)]$

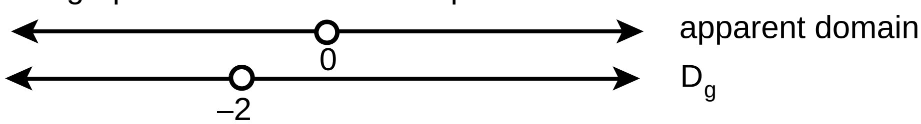
$$f[(g)x] = \frac{2}{\dfrac{x}{x+2}} = \frac{2(x+2)}{x}$$

The domain of the composite **seems to be** $(-\infty,\ 0) \cup (0,\ \infty)$, but we must also consider the domain of g(x).

C. **The actual domain of the composite function is the intersection of the "apparent domain" of the result and the domain of the "inner function**"

$$\left[(-\infty,\ 0) \cup (0,\ \infty)\right] \cap \left[(-\infty,\ -2) \cup (-2,\ \infty)\right]$$

Consider the graphs of the two domains placed one above the other:



The domain of the composite is the **overlap (intersection)** of the two graphs, i.e. $\left[(-\infty,\ 0) \cup (0,\ \infty)\right] \cap \left[(-\infty,\ -2) \cup (-2,\ \infty)\right] = (-\infty,\ -2) \cup (-2,\ 0) \cup (0,\ \infty)$

D. For Practice: Find each of the following compositions and their domains.

1. $(g \circ f)(x)$
2. $(f \circ f)(x)$
3. $(g \circ g)(x)$

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1. $(g \circ f)(x) = \dfrac{1}{1+x}$; its domain is $(-\infty,\ -1) \cup (-1,\ 0) \cup (0,\ \infty)$
2. $(f \circ f)(x) = x$; its domain is $(-\infty,\ 0) \cup (0,\ \infty)$
3. $(g \circ g)(x) = \dfrac{x}{3x+4}$; its domain is $(-\infty,\ -2) \cup \left(-2,\ -\dfrac{4}{3}\right) \cup \left(-\dfrac{4}{3},\ \infty\right)$