USING TRIGONOMETRY

SOLVING A TRIANGLE

"Solving a triangle" means finding the sizes of all angles and sides. To get the most accurate results, use given information whenever possible and do not approximate until the final step.

I. Given two angles and one side (ASA or AAS)

A. Find third angle by subtracting sum of known angles from 180°.
B. Use ***Law of Sines*** to find unknown sides.

II. Given two sides and an angle ***opposite*** one of the known sides (SSA)

A. Use Law of Sines to find angle opposite other known side. (If the sine of the angle is ***greater than one***, then NO triangle can be constructed with given parts.)
B. Subtract the angle found in Part A from 180° to get second possibility. One of the angles found in Parts A and B will be obtuse. Add obtuse angle to given angle.

1. If resulting sum is greater than or equal to 180°, discard the obtuse angle as an option. Use ***Law of Sines*** (or ***Law of Cosines***) to find third side.
2. If resulting sum is less than 180°, do two separate calculations as there are two different triangles possible. Use ***Law of Sines*** (or ***Law of Cosines***) to find each third side.

III. Given two sides and included angle (SAS)

A. Use ***Law of Cosines*** to find third side.
B. Use ***Law of Sines*** (or ***Law of Cosines***) to find ***smaller*** of two unknown angles. This angle will be opposite the shorter side. This angle will always be acute, thus avoiding ambiguity.
C. Subtract sum of the two angles from 180° to find third angle.

IV. Given three sides (SSS)

A. Use ***Law of Cosines*** to find largest angle (opposite largest side). Finding this angle first guarantees that remaining two angles will be acute; no ambiguity exists.
B. Use ***Law of Sines*** (or ***Law of Cosines***) to find a second angle.
C. Subtract sum of the two angles from 180° to find third angle.

V. Given three angles (AAA) - ***not enough information*** to determine a unique triangle.

FINDING AREA OF A TRIANGLE

I. Given one side and altitude drawn to that side. (In a right triangle, the legs constitute a a base and corresponding altitude.)

A = one-half product of base and altitude

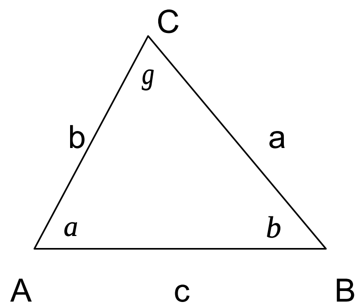
II. Given two sides and ***included*** angle (SAS)

A = one-half product of two sides and ***sine*** of angle between them

III. Given three sides (SSS)

Use Heron's formula (also called Hero's formula)

IV.



Δ ABC is probably not a right triangle, however all of these formulas will work with a right triangle.

LAW OF SINES

I. $\dfrac{\sin a}{a} = \dfrac{\sin b}{b} = \dfrac{\sin g}{c} \Rightarrow \dfrac{a}{\sin a} = \dfrac{b}{\sin b} = \dfrac{c}{\sin g}$

II. Use the two ratios which best match the given information.

III. There is an ambiguity involved when using the Law of Sines to find an angle. See Part II under "Solving a Triangle".

LAW OF COSINES

I. $a^2 = b^2 + c^2 - 2bc(\cos a)$; $b^2 = a^2 + c^2 - 2ac(\cos b)$; $c^2 = a^2 + b^2 - 2ab(\cos g)$

II. $\cos a = \dfrac{b^2 + c^2 - a^2}{2bc}$; $\cos b = \dfrac{a^2 + c^2 - b^2}{2ac}$; $\cos g = \dfrac{a^2 + b^2 - c^2}{2ab}$

III. There is no ambiguity involved when using Law of Cosines to find an angle. The angle found will be unique.

AREA OF A TRIANGLE

I. $A = \dfrac{1}{2}bh$

II. $A = \dfrac{1}{2}bc(\sin a)$; $A = \dfrac{1}{2}ac(\sin b)$; $A = \dfrac{1}{2}ab(\sin g)$

III. Heron's formula: $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = \dfrac{1}{2}$ perimeter