

## GEOMETRIC CONCEPT OF LIMIT

You have been hired by Mr. Vogelsang to put on a fireworks display from the Henley Street bridge. Vogelsang is very patriotic and wants the initial shell to eject a small American flag. You figure that a little remote control device will probably do the job, so you agree. However, it turns out that Vogelsang's ultra-patriotic father is a patient in Baptist hospital, and Vogelsang wants the flag ejected at a height of 80 feet ( $\pm 1$  ft) on the upward leg of the flight so that it will be right in front of his father's window. How much leeway do you have in triggering the ejection device? The problem can be analyzed as follows:

I.  $s(t) = -16t^2 + v_0t + s_0$  gives the **height** of an object at time  $t$ , provided the object is projected **straight upward** (on Earth) or is dropped straight downward [ignoring drag]. This function does not represent the path of the object!

- A. Position is in feet above ground [ideally at sea level]
- B.  $t$  is in seconds after launch
- C.  $-16\text{ft}/\text{sec}^2$  is a constant determined by Earth's gravitational constant
- D.  $v_0$  is the initial velocity in ft/sec
- E.  $s_0$  is the initial position in feet above the ground
- F. The coordinates of the vertex of the parabola  $(t, s(t))$  represent the following:

$t$  = number of seconds needed to attain maximum height;

$s(t)$  = maximum height in feet

$t = -\frac{b}{2a}$  gives the value of  $t$  at the vertex

$s\left(-\frac{b}{2a}\right)$  gives the value of  $s(t)$  at the vertex

II. Suppose that the initial velocity [thrust] of the rocket is 64 ft/sec and that the launch pad is 32 feet above the ground. Then the position function is  $s(t) = -16t^2 + 64t + 32$

A. Check to make certain that the rocket is going to go **at least 80** feet into the air.

1.  $t = -\frac{64}{-32} = 2$  and  $s(2) = -16(4) + 64(2) + 32 = 96$

2. The maximum height of the rocket is 96 feet, so we can continue with the problem.

B. Shell is 80 feet above the ground when position = 80

C.  $80 = -16t^2 + 64t + 32 \Rightarrow t = 1$  sec or 3 sec. We must use  $t = 1$  sec for the upward leg.

D.  $\lim_{t \rightarrow 1} (-16t^2 + 64t + 32) = 80$  because  $s(t)$  is a continuous function. [explained in later section]

The limit represents the height at  $t = 1$  sec.

E. Epsilon-delta definition of limit

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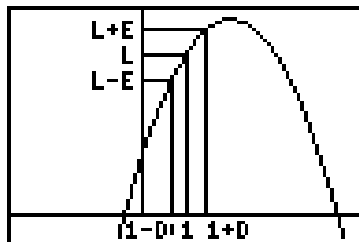
1. In general:  $\lim_{x \rightarrow a} f(x) = L$  if for every number  $\epsilon > 0$  there is a corresponding number  $\delta > 0$  such that  $|f(x) - L| < \epsilon$  whenever  $0 < |x - a| < \delta$ .

2. In this problem:  $\lim_{t \rightarrow 1} s(t) = 80$  if for every number  $\epsilon > 0$  there is a corresponding number  $\delta > 0$  such that  $|f(x) - L| < \epsilon$  whenever  $0 < |x - a| < \delta$ .

III. How much can  $t$  vary and still eject the flag within 1 foot of the desired height of 80 feet? This means that  $\epsilon = 1$  in this problem.

A. Graph of the position [height] function

Note: This is NOT the path of the rocket.



B.  $\epsilon = 1 \Rightarrow L + \epsilon = 81$

1.  $81 = -16t^2 + 64t + 32 \Rightarrow t = 1.03175$
2.  $1 + \delta = 1.03175 \Rightarrow \delta = 0.03175 \text{ sec}$

C.  $\epsilon = 1 \Rightarrow L - \epsilon = 79$

1.  $79 = -16t^2 + 64t + 32 \Rightarrow t = 0.96922$
2.  $1 - \delta = 0.96922 \Rightarrow \delta = 0.03078 \text{ sec}$

D. To ensure you are within the required interval, always choose  $\delta = \text{SMALLER}$  value, so  $\delta = 0.03078 \text{ sec}$ .

E. In order to have the flag ejected at  $80 \pm 1$  feet, the ejection must be triggered at  $t = 1 \pm 0.03078$  seconds.

IV. Epsilon-delta definition:  $|(-16t^2 + 64t + 32) - 80| < 1$  whenever  $0 < |t - 1| < 0.03078$ .