NUMERICAL INTEGRATION

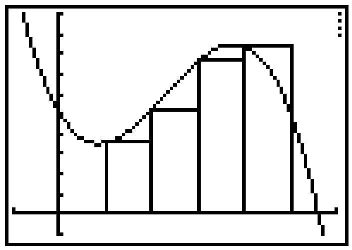
Objective: Approximate integrals using trapezoidal and Simpson's rules

Numerical integration can be useful

- If it is impossible (for us, at least) to integrate the function
- If there is no formula for the function

Previous methods of approximate integration using rectangles: left, right, and midpoint

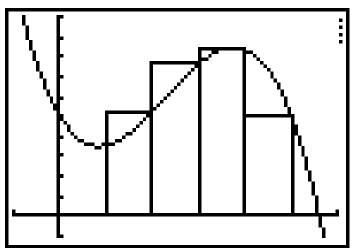
- Left rectangles:



Area under the graph of y = f(x) from x = 1 to x = 5 using 4 subintervals: $\Delta x = 1$

$A \approx L_4 = [f(1) + f(2) + f(3) + f(4)](1) = 24.8$

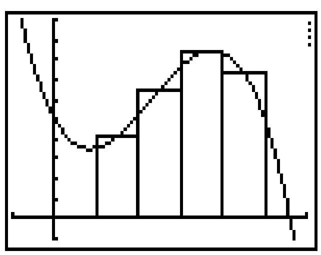
- Right rectangles:



Area under the graph of y = f(x) from x = 1 to x = 5 using 4 subintervals: $\Delta x = 1$

$A \approx R_4 = [f(2) + f(3) + f(4) + f(5)](1) = 26.24$

- Midpoint rectangles:



Area under the graph of y = f(x) from x = 1 to x = 5 using 4 subintervals: $\Delta x = 1$

$A \approx M_4 = [f(1.5) + f(2.5) + f(3.5) + f(4.5)](1) = 26.36$

Use left, right, and midpoint rectangles to approximate $\int_0^3 \frac{1}{1+y^5}\,dy$ using 6 rectangles. Round answers to 6 decimal places.

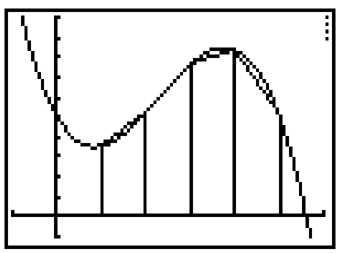
Using trapezoids instead of rectangles allows the figures to more closely fit under the curve.

- Area of a trapezoid = $\frac{1}{2}h(b_1 + b_2)$
- Trapezoidal rule: $\int_a^b f(x)dx =$

$$\frac{1}{2}f(x_0 + x_1)\Delta x + \frac{1}{2}f(x_1 + x_2)\Delta x + \frac{1}{2}f(x_2 + x_3)\Delta x + \ . \ . \ . \ + \frac{1}{2}f(x_{n-2} + x_{n-1})\Delta x + \frac{1}{2}f(x_{n-1} + x_n)\Delta x$$

$$\int_a^b f(x)dx \approx T_n = \frac{\Delta x}{2}[f(x_0) + 2f(x_1) + 2f(x_2) + \ldots 2f(x_{n-1}) + f(x_n)]$$

- Trapezoids:



Area under the graph of $y = f(x)$ from $x = 1$ to $x = 5$ using 4 subintervals: $\Delta x = 1$

$$A \approx T_4 = \frac{1}{2}(1)\{[f(1) + f(2)] + [f(2) + f(3)] + [f(3) + f(4)] + [f(4) + f(5)]\} = 25.52$$

or $A \approx T_4 = \frac{1}{2}[f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)] = 25.52$

NOTE: $T_4 = \frac{1}{2}(L_4 + R_4)$! $\qquad \frac{1}{2}(24.8 + 26.24) = 25.52$

Use trapezoids with n = 6 to approximate $\int_0^3 \frac{1}{1+y^5}dy$ to 6 decimal places.

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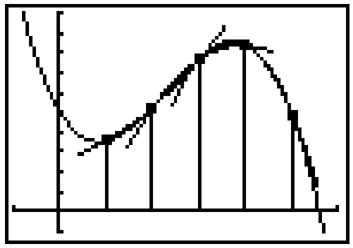
Simpson's rule is the most accurate approximation method; it uses portions of parabolas for the upper boundaries.

- Simpson's



Area under the graph of $y = f(x)$ from $x = 1$ to $x = 5$ using 4 subintervals: $\Delta x = 1$

$$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \ldots 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

- $A \approx S_4 = \frac{1}{3}[f(1) + 4f(2) + 2f(3) + 4f(4) + f(5)] = 26.08$
- n must be ≥ 4 and must be even
- Note that $S_{2n} = \frac{1}{3}T_n + \frac{2}{3}M_n$

Use Simpson's Rule with n = 6 to approximate $\int_0^3 \frac{1}{1+y^5}dy$ to 6 decimal places.

Use fnInt to approximate $\int_0^3 \frac{1}{1+y^5}dy$ to 6 decimal places.

Solutions

$$A \approx L_6 = \frac{1}{2}\left[f(0) + f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right)\right] = 1.313250$$

$$A \approx R_6 = \frac{1}{2}\left[f\left(\frac{1}{2}\right) + f(1) + f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right) + f(3)\right] = 0.815299$$

$$A \approx M_6 = \frac{1}{2}\left[f\left(\frac{1}{4}\right) + f\left(\frac{3}{4}\right) + f\left(\frac{5}{4}\right) + f\left(\frac{7}{4}\right) + f\left(\frac{9}{4}\right) + f\left(\frac{11}{4}\right)\right] = 1.067416$$

$$A \approx T_6 = \frac{1}{4}\left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + 2f(2) + 2f\left(\frac{5}{2}\right) + f(3)\right] = 1.064275$$

$$A \approx S_6 = \frac{1}{6}\left[f(0) + 4f\left(\frac{1}{2}\right) + 2f(1) + 4f\left(\frac{3}{2}\right) + 2f(2) + 4f\left(\frac{5}{2}\right) + f(3)\right] = 1.074915$$

fnInt $\approx$ 1.065879