

DIFFERENTIAL EQUATIONS

7.1 Modeling with Differential Equation

Objective: Solve differential equations

I. Differential equation

- A. An equation which contains an unknown function and one or more of its derivatives
- B. Order of a differential equation is the order of the highest derivative that occurs in the equation
- C. f is a solution of a differential equation if the equation is satisfied when $y = f(x)$ and its derivatives are substituted into the equation

II. General and particular solutions

- A. Show that every member of the family of functions $y = Cte^{-t}$ is a solution of the second order differential equation

$$y'' + 2y' + y = 0 \quad \text{[general solution]}$$

$$1. y' = (Ct)(e^{-t}(-1)) + e^{-t}C = -Cte^{-t} + Ce^{-t}$$

$$2. y'' = -(Ct)(e^{-t}(-1)) + e^{-t}(-C) + Ce^{-t}(-1) \\ = Cte^{-t} - 2Ce^{-t}$$

$$3. Cte^{-t} - 2Ce^{-t} - 2Cte^{-t} + 2Ce^{-t} + Cte^{-t} = 0 \quad \mathbf{Y} \quad 0 = 0$$

- B. Find a particular solution of the above equation if $y(1) = e$

$$1. y(1) = e = Ce^{-1} \quad \mathbf{Y} \quad C = e^2$$

$$2. y = e^2te^{-t} \text{ is a particular solution which satisfies the initial condition}$$