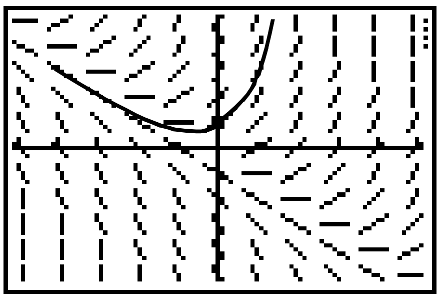
DIFFERENTIAL EQUATIONS

7.2 Direction Fields and Euler's Method

Objective: Draw the solution curve of an initial-value differential equation

I. Using a slope field (direction field) to sketch a curve

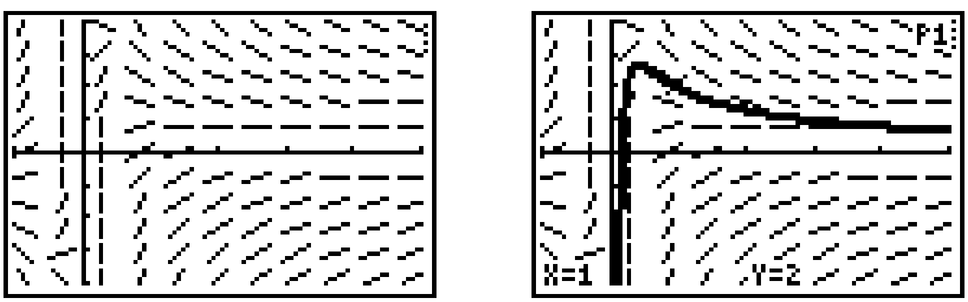
A. Sketch the graph of y if $y' = x + y$ and y(0) = 1 [slope at (0, 1) is 1]

B. 

II. Using a slope field to check a solution

A. Verify graphically that $y = \dfrac{2 + \ln x}{x}$ is a solution of the initial-value problem

$x^2 y' + xy = 1,\ y(1) = 2$

B. 

III. See Example 1 on p. 514 for instructions on how to draw a slope field by hand

IV. If the independent variable is missing from the right side, the differential equation is said to be autonomous

A. $\dfrac{dI}{dt} = 15 - 3I$

B. The slopes corresponding to two different points with the same y-coordinate must be equal

C. Graphs of solutions are horizontal translations of each other.

D. See Example 2 on p. 515

V. Go to the following URL for a slope field generator:

http://alamos.math.arizona.edu/ODEApplet/JOdeApplet.html

A. $y' = 1 - xy$

B. (0, 0), (-.5, 1.5), (-.5, -2.5)

VI. Euler's Method for numerical approximations to solutions of differential equations

Genreal formula: $y_n = y_{n-1} + hF(x_{n-1}, y_{n-1})$

Use Euler's method with step size 0.2 to estimate y(1) if $y' = x + y$ with $y(0) = 1$

1. We are given h = 0.2, $x_0 = 0$, $y_0 = 1$
2. Let $y_0 = 1$ be the initial guess — $y(1) \approx 1$
3. A better guess is: $y_1 = y_0 + 0.2F(x_0, y_0)$ = 1 + 0.2(0 + 1) = 1.2 — $y(1) \approx 1.2$
4. Even better guess is: $y_2 = y_1 + 0.2F(x_1, y_1)$ = 1.2 + 0.2(0.2 + 1.2) — $y(1) \approx 1.48$
5. Better guess: $y_3 = y_2 + 0.2F(x_2, y_2)$ = 1.48 + 0.2(0.4 + 1.48) — $y(1) \approx 1.856$
6. Better guess: $y_4 = y_3 + 0.2F(x_3, y_3)$ = 1.856 + 0.2(0.6 + 1.856) — $y(1) \approx 2.3472$
7. Better guess: $y_5 = y_4 + 0.2F(x_4, y_4)$ = 2.3472 + 0.2(0.8 + 2.3472) — $y(1) \approx 2.97664$
8. Last guess: $y_6 = y_5 + 0.2F(x_5, y_5)$ = 2.97664 + 0.2(1 + 2.97664) — $y(1) \approx 3.0561728$