

DIFFERENTIAL EQUATIONS

7.4 Separable Equations

Objective: Solve differential equations symbolically

I. A separable equation is a first-order differential equation which can be written in the form

$$\frac{dy}{dx} = g(x)f(y)$$

II. Solve the differential equation $y' = \frac{6x^2}{2y + \cos y}$

A. Write using Leibniz notation: $\frac{dy}{dx} = \frac{6x^2}{2y + \cos y}$

B. Rewrite in differential notation: $(2y + \cos y)dy = 6x^2 dx$

C. Integrate both sides: $\int (2y + \cos y)dy = \int 6x^2 dx$

$$y^2 + \sin y = 2x^3 + C$$

D. Solve for y if possible.

II. Solve the above equation with the initial condition $y(1) = p$

A. Substitute into $y^2 + \sin y = 2x^3 + C$ to get $p^2 + \sin p = 2(1)^3 + C \Rightarrow C = p^2 - 2$

B. The specific solution is $y^2 + \sin y = 2x^3 + p^2 - 2$

III. Omit orthogonal trajectories