THE DERIVATIVE

Review Notes on Differentiation for MATH. 1920 with Practice Exercises

The Leibnitz notation for the derivative of f(x) with respect to x is $\frac{d}{dx}f(x)$. It is commonly used whenever we are asked to find the derivative, but do not have the function given as an equation. It is essential whenever it is not clear which letter in a function is the independent variable. It was used to introduce every differentiation rule in MATH 1910:

See pages 192-6,198,202,205, 223-224,228-234,240,241,243,244,247-51,268,270, etc.

Several different notations for the first derivative are shown on p. 162. Higher order derivatives and their notations are discussed on p. 165-8.

Notation is very important because poor notation misleads others who try to follow your work, and it frequently misleads you as well. Do not put equal signs between quantities which are not equal. The symbol $\frac{d}{dx}$ has no meaning because we do not know what to differentiate!

Find and <u>simplify</u> each of the following derivatives.

1. $\frac{d}{da}(ax^2y^3 - a^4bc + p)$ HINT: The variable is “a”; treat all other letters as constants.

2. $\frac{d^2}{db^2}(ax^2y^3 - a^4bc + p)$ 3. $\frac{d}{dy}(ax^2y^3 - a^4bc + p)$ 4. $\frac{d}{dx}(ax^2y^3 - a^4bc + p)$

Some derivatives can be found using different methods. Whenever you are tempted to use a cumbersome method such as the product or quotient rule, study the problem to see if an easier approach will work. [See note at bottom of p. 205.] No matter which method you use, you must simplify the answer!

Find and <u>simplify</u> each of the following derivatives, using the specified methods. [Sec. 3.1]

5. $\frac{d}{dx}\left(\frac{x^2 + 4x + 3}{\sqrt{x}}\right)$ Use the quotient rule first. Then divide the denominator into the numerator and rework the problem using the power rule. Show that your answers are equivalent.

6. $\frac{d}{dt}\sqrt[3]{t}(t + 2)$ Use the product rule first. Then distribute the monomial and rework the problem using the power rule. Show that your answers are equivalent.

Many differentiation problems require the use of the chain rule. The chain rule is never used alone, but is always used in combination with at least one other rule. Some situations requiring the chain rule are quantities raised to powers, roots of quantities, trig functions of quantities, exponents which are quantities, etc. Sometimes other methods can be employed, but the chain rule is usually easier and shorter. The answer needs to be in factored form.

Find and <u>simplify</u> each of the following, using the specified methods on #7. [Sec. 3.5]

7. $\frac{d}{dx}(3x-7)^2$ Expand the quantity and use the power rule. Then rework the problem using the chain rule.

8. $\frac{d}{dx}(3x-2)^{10}(5x^2-x+1)^{12}$

9. $\frac{d}{dy}\left(\frac{y^3+1}{y^3-1}\right)$

Trig functions and log functions must have arguments. The words "sin", "tan", "log" have meaning in the English language but no mathematical meaning. The tendency in these problems is to use the product rule where it does not apply.

Find and simplify each of the following derivatives. [Sec. 3.1,3.4, 3.5, 3.7]

10. $\frac{d}{dx}\cos(\tan x)$

11. $\frac{d}{dx}\sin[\cos(\tan 2x)]$

12. $\frac{d}{dx}\tan^3(-ax^2)$

13. $\frac{d}{dx}e^{x\cos x}$

14. $\frac{d}{dx}\left(x^2e^{-x}\right)$

15. $\frac{d}{dx}\log_3(x^2-4)$

16. $\frac{d}{dx}5^{-1/x}$

17. $\frac{d}{dx}\ln\left|xe^{-x}\right|$

It is essential that you learn to apply formulas for both derivatives and integrals because most problems are too complicated to do easily by hand. While you do not have to memorize the rules for differentiating the inverse trig functions, you must be able to apply them.

Find and simplify each of the following derivatives. [Sec. 3.6]

18. $\frac{d}{dx}\tan^{-1}(e^{4x})$

19. $\frac{d}{dr}\sin^{-1}(x^2+2)$

20. Hyperbolic functions are discussed on p. 253-4. Use the definitions given there to show that $\frac{d}{dx}(\sinh x)=\cosh x$. This is very similar to which other trig derivative?

Limits are very important in calculus because it is from limits that we get derivatives. While numerical tables and graphs may be helpful in some instances, they can be very misleading in others. Therefore, they should not be used as the sole means of finding a limit. There are several classic techniques taught in Algebra II which are commonly used in finding limits. These techniques include factoring, rationalizing the *numerator*, and simplifying compound fractions by multiplying both numerator and denominator by the LCD of the small fractions in the numerator of the compound fraction. Remember that $\frac{0}{0}$ is an **indeterminate** form and does not equal anything! However, if you get $\frac{0}{0}$ when you try to use direct substitution to find a limit, it means there probably is a limit. It is your job to rewrite the problem in such a way that you can find its limit! The limit of a function will fail to exist at $x = a$ if the function has a jump or infinite discontinuity at $x = a$, or if the right-hand and left-hand limits as x approaches a are not equal. A function does NOT have to be defined at $x = a$ to have a limit there!

Find and simplify each of the following limits. [Sec. 2.3, 2.8]

21. $\lim_{x \to 0} \frac{\sqrt{5 - x} - \sqrt{5}}{x}$ 22. $\lim_{h \to 0} \frac{(3 + h)^{-1} - 3^{-1}}{h}$ 23. $\lim_{x \to 2} \frac{x^4 - 16}{x - 2}$

The absolute value function is a piece-wise defined function and finding limits of functions involving absolute value requires special attention. [Exception: logarithmic functions] A classic limit involving absolute value is $\lim_{x \to a} \frac{|x - a|}{x - a}$. Although the right-hand and left-hand limits exist, they are not equal which means that the limit of the function as x approaches a does not exist. The graph is a good guide in this type of problem, however you must be able to show algebraically that the limit does not exist! Study my notes on this section.

Find and simplify each of the following limits. If a limit does not exist, explain why based on algebraic work. [Sec. 2.8]

24. $\lim_{x \to 2^+} \frac{|x - 2|}{x - 2}$ 25. $\lim_{x \to -5^-} \frac{|x + 5|}{x + 5}$ 26. $\lim_{x \to 9/2} \frac{|9 - 2x|}{9 - 2x}$

There is a special technique which can be used for finding the limit of the quotient of two polynomials as $x \to \pm\infty$. Divide each term in both numerator and denominator by the largest power of the variable in the *denominator* and then apply the fact that $\lim_{x \to \pm\infty} \frac{1}{x^n} = 0$, provided n is a positive integer. There are three possible types of limits for these problems: 0, a nonzero real number, or $\pm\infty$.

Find and simplify each of the following limits without using l'Hospital's Rule. [Sec. 2.5]

27. $\lim_{x \to \infty} \frac{7x^3 + 4x}{2x^3 - x^2 + 3}$ 28. $\lim_{x \to -\infty} \frac{7x^4 + 4x}{2x^3 - x^2 + 3}$ 29. $\lim_{x \to \infty} \frac{7x^3 + 4x}{2x^4 - x^2 + 3}$

If, when you try direct substitution to find a limit, you get an indeterminate form of $\frac{0}{0}$ or $\pm\frac{\infty}{\infty}$ you may use l'Hospital's Rule. Sometimes it is necessary to rewrite the function in order to obtain one of the required indeterminate forms. Note that although these functions involve quotients you do NOT use the quotient rule. We will have need of l'Hospital's Rule later in this course. Write the indeterminate form above the problem and use $\overset{H}{=}$ to show that you have applied l'Hospital's Rule. Always SIMPLIFY before you try to apply the rule again!

Find and simplify each of the following limits. [Sec. 4.5]

30. $\lim_{x \to \infty} \frac{7x^3 + 4x}{2x^3 - x^2 + 3}$ 31. $\lim_{x \to \infty} (4x^3 e^{x^3})$