SUMMARY OF CONVERGENCE AND DIVERGENCE TESTS FOR SERIES AND SEQUENCES

Sec	Test	Series	Convergence or Divergence	Comments
2	Geometric Series	$\sum_{n=1}^{\infty} ar^{n-1}$	i) Converges if $\lvert r \rvert < 1 \Rightarrow$ sum $S = \frac{a}{1-r}$. ii) Diverges if $\lvert r \rvert \geq 1$.	Useful for ***comparison*** tests if the nth term a_n of a series is ***similar*** to ar^{n-1}.
2	Test for Divergence	$\sum a_n$	Diverges if $\lim_{n\to\infty} a_n \neq 0$.	Inconclusive if $\lim_{n\to\infty} a_n = 0$. Also called nth-term test.
3	p-Series Test	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	i) Converges if $p > 1$. ii) Diverges if $p \leq 1$.	Useful for ***comparison*** tests if the nth term a_n of a series is ***similar*** to $\frac{1}{n^p}$.
3	Integral Test	$\sum_{n=1}^{\infty} a_n$, $a_n = f(n)$	i) Converges if $\int_1^{\infty} f(x)dx$ converges. ii) Diverges if $\int_1^{\infty} f(x)dx$ diverges.	The function f obtained from $a_n = f(n)$ must be **continuous, positive, decreasing**, and readily integrable.
3	Comparison Test	$\sum a_n$, $\sum b_n$, $a_n > 0$, $b_n > 0$	i) If $\sum b_n$ converges and $a_n \leq b_n$ for every n, then $\sum a_n$ converges. ii) If $\sum b_n$ diverges and $a_n \geq b_n$ for every n, then $\sum a_n$ diverges.	The comparison series $\sum b_n$ is often a ***geometric series*** or a ***p-series.***
3	Limit Comparison Test	$\sum a_n$, $\sum b_n$ $a_n > 0$, $b_n > 0$	If $\lim_{n\to\infty} \frac{a_n}{b_n} = c$, for some ***positive*** real number c, then both series converge or both diverge.	To find b_n consider only the terms of a_n that have the greatest effect on the magnitude.
4	Ratio Test	$\sum a_n$	If $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = L$, then the series i) converges (absolutely) if $L < 1$. ii) diverges if $L > 1$ (or $= \infty$).	Inconclusive if $L = 1$. Useful if a_n involves ***factorials*** or ***nth powers***. If $a_n > 0$ for every n, the absolute value sign may be disregarded.
4	Alternating Series Test	$\sum (-1)^n b_n$ $b_n > 0$	Converges if $b_{n+1} \leq b_n$ for every n and $\lim_{n\to\infty} b_n = 0$.	Applies ***only*** to alternating series.
4	Absolute convergence	$\sum a_n$	If $\sum \lvert a_n \rvert$ converges, then $\sum a_n$ converges.	Useful for series that contain both positive and negative terms.

Sec	Test	Sequence	Convergence or Divergence	Comments
1	Associated Function Test	a_n	If $\lim_{n\to\infty} f(x) = L$ and $f(n) = a_n$ when n is an integer, then $\lim_{n\to\infty} a_n = L$	Useful when l'Hôpital's rule applies to the function f(x).
1	Fraction Test	$\frac{1}{n^r}$	$\lim_{n\to\infty} \frac{1}{n^r} = 0$ if $r > 0$.	The base n in the denominator is raised to a fixed power r.
1	Sandwich (Squeeze) Theorem	b_n	If $a_n \leq b_n \leq c_n$ for $n \geq n_0$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$, then $\lim_{n\to\infty} b_n = L$.	L must be a ***real*** number.
1	Absolute Value Test	$(-1)^n a_n$	If $\lim_{n\to\infty} \lvert a_n \rvert = 0$, then $\lim_{n\to\infty} a_n = 0$.	Applies only to ***alternating*** sequences.
1	Power Test	r^n	The sequence $\{r^n\}$ is convergent if $-1 < r \leq 1$ and divergent for all other values of r. $\lim_{n\to\infty} r^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$	The fixed base r is raised to the nth power.