

MTH 1420 SUMMATIONS WORKSHEET

The following summation formulas can be proved using mathematical induction; see handout on my web page for explanation and examples.

Assume that: $\sum_{k=1}^n c = nc$, $\sum_{k=1}^n k = \frac{n(n+1)}{2}$, $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$, $\sum_{k=1}^n k^3 = \frac{n^2(n+1)^2}{4}$

1. Write out each of the following sums and then use the formulas above to evaluate each. Show the first 5 terms and the last term. Write out all details as shown in this example.

Example:
$$\sum_{k=7}^{20} (2k - 3) = 11 + 13 + 15 + 17 + 19 + \dots + 37$$

$$\begin{aligned} \sum_{k=7}^{20} (2k - 3) &= \sum_{k=1}^{20} (2k - 3) - \sum_{k=1}^6 (2k - 3) \\ &= 2 \sum_{k=1}^{20} k - \sum_{k=1}^{20} 3 - 2 \sum_{k=1}^6 k + \sum_{k=1}^6 3 \\ &= 2 \frac{20(21)}{2} - 20(3) - 2 \frac{6(7)}{2} + 6(3) = 336 \end{aligned}$$

a. $\sum_{k=5}^{25} (3k + 2)$ b. $\sum_{k=1}^{100} (2k^2 - 4k^3)$ c. $\sum_{m=5}^{200} (3 - 2m)$ d. $\sum_{j=1}^{2n} (3 - 2j^2)$

2. Fill in the correct values and/or expressions for the question marks in order to maintain equality of each pair of sums. Show the check for the first 3 terms and the last term as shown in this example. [HINT: look for a pattern involving "shifting"]

Example: $\sum_{k=1}^n (3k) = \sum_{k=?}^{??} 3(k+1)$ Answer: Replace ? by 0 and ?? by (n - 1)

Check: When $k = 1$, $3k = 3(1) = 3$ and when $k = 0$, $3(k+1) = 3(0+1) = 3(1) = 3$
 When $k = 2$, $3k = 3(2) = 6$ and when $k = 1$, $3(k+1) = 3(1+1) = 3(2) = 6$
 When $k = 3$, $3k = 3(3) = 9$ and when $k = 2$, $3(k+1) = 3(2+1) = 3(3) = 9$
 When $k = n$, $3k = 3(n) = 3n$ and when $k = (n-1)$, $3(k+1) = 3[(n-1)+1] = 3(n) = 3n$

a. $\sum_{k=1}^n (3k) = \sum_{k=5}^{n+4} 3(??)$ b. $\sum_{k=4}^{100} k^3 = \sum_{k=?}^{??} (k+2)^3$

c. $\sum_{k=4}^{100} k^2 = \sum_{k=?}^{??} (k-5)^2$ d. $\sum_{k=4}^{100} (k+3)^2 = \sum_{k=0}^{96} (??)^2$

3. Compute each of the following sums directly, showing your work.

a. $\sum_{k=1}^5 (-1)^k$

b. $\sum_{k=1}^8 (-1)^k$

c. $\sum_{k=1}^{10} (-1)^k$

d. $\sum_{k=1}^{13} (-1)^k$

e. $\sum_{k=0}^5 (-1)^k$

f. $\sum_{k=6}^{14} (-1)^k$

g. $\sum_{k=3}^{15} (-1)^k$

h. $\sum_{k=5}^{14} (-1)^k$

4. Find a general [piece-wise] formula for $\sum_{k=1}^n (-1)^k$.

5. Find a general [piece-wise] formula for $\sum_{k=a}^b (-1)^k$.

6. Consult your user's guide to learn how to do summations on your calculator. Show your screen display for finding this summation.

Find $\sum_{k=1}^{100} (2k^2 - 4k^3)$

Caution! This is not a substitute for showing your work on problems, but it is great for checking your answers.