

Taylor and Maclaurin Series

$$\sum_{x=0}^4 \frac{1}{x!} = \text{sum}(\text{seq}(1/x!, x, 0, 4, 1)) \approx 2.70833$$

$$D_x \tan^{-1}(x) = \frac{1}{1+x^2}$$

Power Series formula for the Geometric Series:

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n \quad | = (-1, 1)$$

Power Series formula for the Taylor Series centered at a:

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Power Series formula for the Maclaurin Series (centered at 0):

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

1. a) Find $M_5(x)$ for $f(x) = e^x$ b) Find the summation formula for the Maclaurin Series for $f(x)$
 c) Find the open interval of convergence d) Use $M_5(x)$ to approximate the value of e (to places) e) How large is the error?

2. a) Find $T_6(x)$ for $f(x) = \ln(x)$ centered at 2 b) Find the summation formula for the Taylor $f(x) = \ln(x)$ centered at 2 c) Find the open interval of convergence d) Use $T_6(x)$ to approximate the value of $\ln(e)$ (to 5 decimal places) e) How large is the error?

3. a) Find $M_5(x)$ for $f(x) = \frac{1}{1+x^2}$ b) Find the summation formula for the Maclaurin series for $f(x) = \frac{1}{1+x^2}$ c) Find the open interval of convergence d) Use $M_5(x)$ to approximate the value of $\frac{2}{3}$ (to 5 decimal places). HINT: Use substitution in a known series.

4. a) Find $M_5(x)$ for $f(x) = \tan^{-1}(x)$ b) Find the summation formula for the Maclaurin Series for $f(x) = \tan^{-1}(x)$ c) Find the open interval of convergence d) Use $M_5(x)$ to approximate the value of $\frac{\pi}{4}$ (to 5 decimal places) d) How large is the error? HINT: Use substitution and integration.

5. a) Find $T_5(x)$ for $f(x) = \sin(x)$ centered at $\frac{\pi}{6}$ b) Find the summation formula for the Taylor Series for $f(x) = \sin(x)$ centered at $\frac{\pi}{6}$ c) Use $T_5(x)$ to find $\sin(31^\circ)$ (to 8 decimal places) d) How large is the error? HINT: Begin with the power series for $\cos(x)$ and use differentiation.