MATH 1910
Section 2.10 What do $f'(x)$ and $f''(x)$ say about $f(x)$?

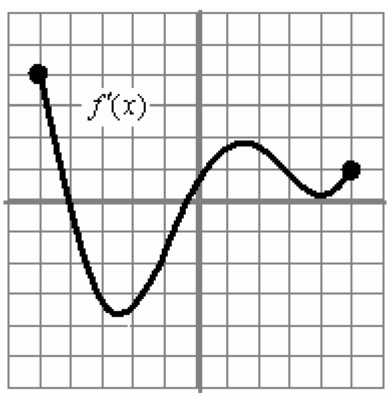
If $f'(x) > 0$ on an interval, then the graph of $f(x)$ is increasing on that interval.
If $f'(x) < 0$ on an interval, then the graph of $f(x)$ is decreasing on that interval.

Ex) The graph of $f'(x)$ is shown below. Determine the following about the graph of $f(x)$:

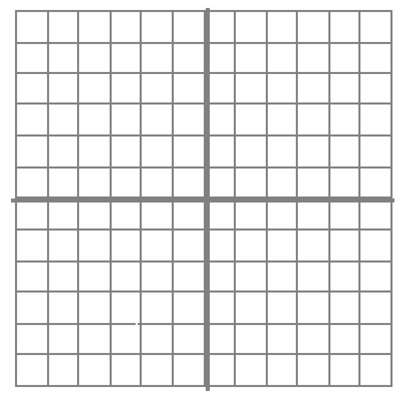
a) On what intervals is $f(x)$ increasing or decreasing?

b) At what values does $f(x)$ have a maximum or a minimum?

c) If it is known that $f(-5) = 1$, sketch a possible graph of $f(x)$.



$f(x)$

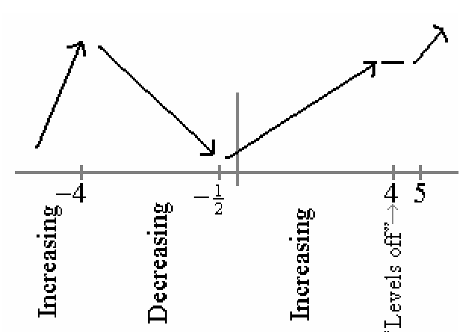


SOLUTION:

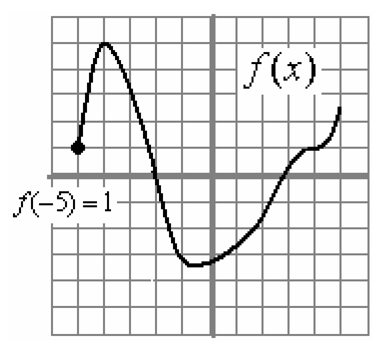
a) $f(x)$ increases on the intervals $(-5, -4)$ and $(-1/2,\ 5)$ and "levels off" at x = 4.
$f(x)$ decreases on the interval $(-4,\ -1/2)$

b) Since $f(x)$ increases left of $x = -4$ and decreases to the right of $x = -4$ … there is a local maximum value at $x = -4$, and $f(x)$ decreases to the left of $x = -1/2$ and increases to the right … there is a local minimum value at $x = -1/2$

c) Using the information from part (a), you can get a rough idea of what the graph of $f(x)$ looks like …



… but without knowing the **initial condition** $f(-5) = 1$ we wouldn't know to start the graph →



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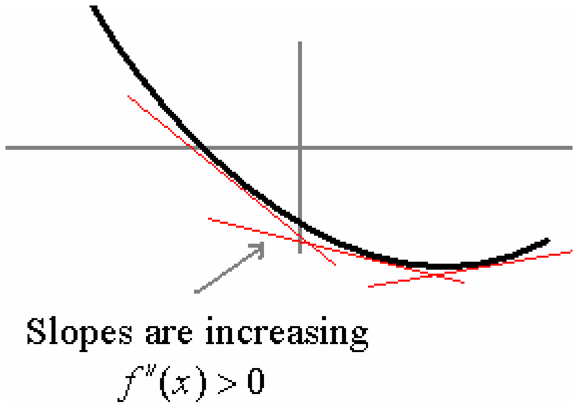
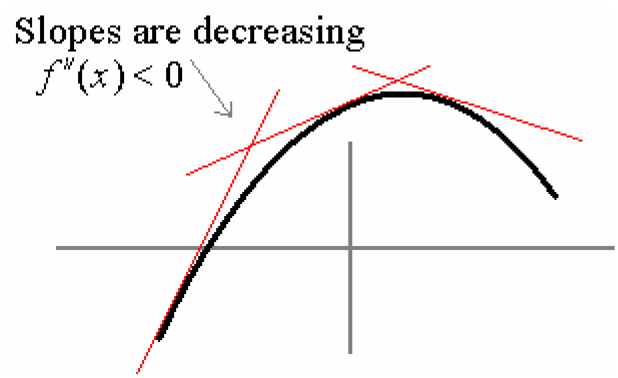
The second derivative, $f''(x)$, details the rate of change in the first derivative … i.e. the rate of change of the "slope" of $f(x)$.





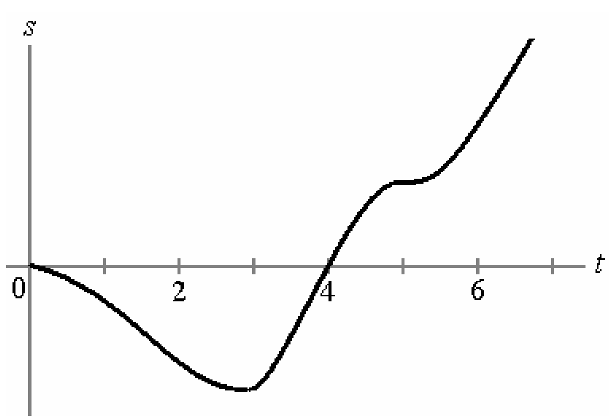
When $f''(x) > 0$ the graph of $f(x)$ is **concave up**

When $f''(x) < 0$ the graph of $f(x)$ is **concave down**

Also, regarding $f''(x)$ as the rate of change in the values of $f'(x)$,

a) When the graph of $f(x)$ is relatively "straight" the values of $f'(x)$ don't change as rapidly … so $f''(x)$ is close to 0.
b) When the graph of $f(x)$ is "curvy" the values of $f'(x)$ change rapidly … so $f''(x)$ has a high magnitude and its sign is indicated by the concavity of the graph of $f(x)$

Ex) A particle is moving along a straight line and the graph below represents its position to the right of its starting point.



a) When is the particle moving left?

b) When is the particle moving right?

c) What happens to the particle at 5 seconds?

d) What are the inflection points of the graph?
Interpret their physical significance.

e) When does the particle have positive acceleration?

f) When is the particle slowing down?

SOLUTION:
a) The graph going "down" should be interpreted as the particle moving left of its starting point. It moves left for the first 3 seconds
b) It moves right during the times the graph starts to go "up" or increase. The particle moves right starting at 3 seconds
c) Since the graph has a horizontal tangent at 5 seconds, the particle temporarily "stops" there and then continues to move right.
d) The inflection points are at 2 seconds, 4 seconds and 5 seconds (approximately) … remember, these are the places when the graph changes concavity. These points represent the times when the velocity reaches a maximum magnitude
e) Positive acceleration occurs between 2 seconds and 4 seconds and after 5 seconds.
f) The particle is "slowing down" between 2 seconds and 3 seconds and between 4 seconds and 5 seconds

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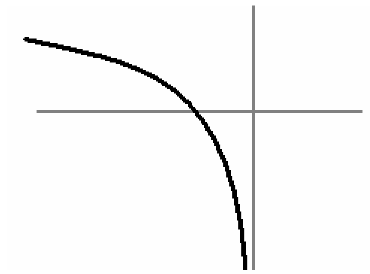
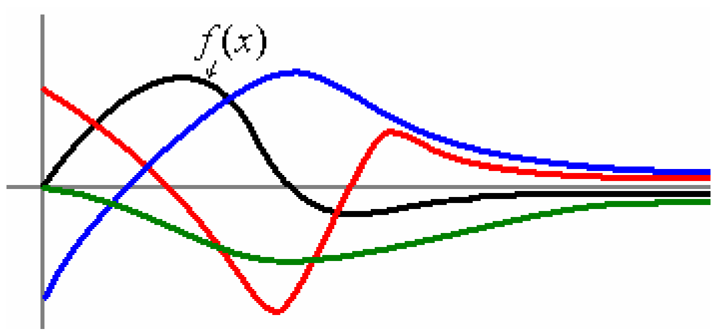
Ex) Sketch a graph of a function whose first derivative and second derivative are always negative.
Could you come up with a formula for such a function?

SOLUTION: $f(x) = \ln(-x)$ is an example … could you think of another?



← Slopes are all negative … and graph is always concave down.

Ex) The graph of a function $f(x)$ is shown below in black.
Which of the graphs here represents the **antiderivative** of $f(x)$?
In other words, which of these other graphs would use the graph of $f(x)$ as its derivative?



SOLUTION: Done in class