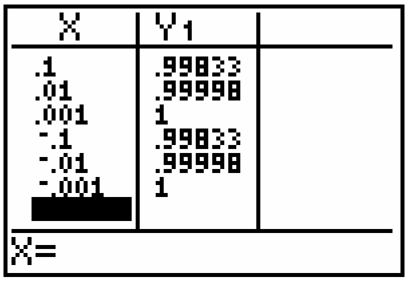
MATH 1910
Section 2.2 The Limit of a Function

Ex. What do the y-values of the graph of $f(x) = \frac{\sin x}{x}$ approach as the x-values approach 0?

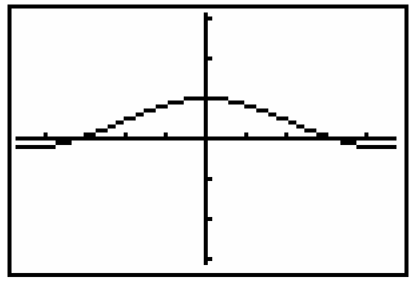
Look at a table of values for the function as x → 0 (use ASK mode)



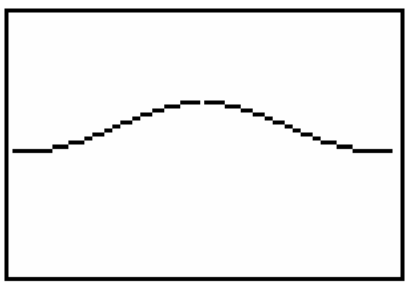
X	Y1
.1	.99833
.01	.99998
.001	1
-.1	.99833
-.01	.99998
-.001	1

X=

Look at the graph as x → 0 (ZOOMDEC)



Based on the numerical data and the graphical data the graph seems to approach y = 1 as x → 0.
But, the function is not defined at x = 0.
Look at the graph with the axes turned off. (2^{nd} FORMAT, AxesOff)



You can barely tell, but there is a hole in the graph at x = 0. The function is not defined there.

<u>DEFINITION OF LIMIT</u>: We write

$\lim_{x \to a} f(x) = L$ and say "the limit of *f(x)* as *x* approaches *a,*equals *L*"

if we can make the values of *f(x)* arbitrarily close to L (as close to L as we like) by taking x to be sufficiently close to equal a .

When evaluating limits, you're actually finding the "intended value" of the function as it approaches a certain x-value.

In the above example the function $f(x) = (\sin x)/x$ is not defined at x = 0, but we can say that the limit as x approaches 0 does equal 1 because that is the "intended" y-value the graph approachs.

So $\lim_{x \to 0} \frac{\sin x}{x}$ does exist and $\lim_{x \to 0} \frac{\sin x}{x}$ = 1, regardless of whether or not the function is defined there.

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Ex. Evaluate these limits either graphically or numerically (using your table):

a) $\lim_{x\to 2} 2x^2 - 3x + 5$ b) $\lim_{x\to -1} \frac{x^2 - 3x - 4}{x+1}$

SOLUTION:

a) there's no trouble just plugging in the value x = 2 into the function. This would give us the value of the limit as x → 2. You should get a limit of L = 7.

b) if you consider a table of values, you notice you can't simply plug in x = – 1 because the function isn't defined there, but as the table suggests, the y-values approach a limit of – 5, so

$$\lim_{x\to -1} \frac{x^2 - 3x - 4}{x+1} = -5$$

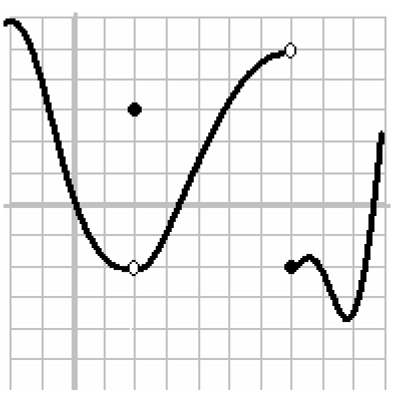
Left hand and Right hand limits: When using your table to evaluate these limits you've considered values to the left and to the right of the central x value you're approaching. These are called the left hand and right hand limits of the function *f (x)*.

$\lim_{x\to a^-} f(x) = L$ (taking values of x approaching a from the left, x < a)

$\lim_{x\to a^+} f(x) = L$ (taking values of x approaching a from the right, x > a)

IMPORTANT! → The limit $\lim_{x\to a} f(x) = L$ exists ONLY if $\lim_{x\to a^-} f(x) = L$ AND $\lim_{x\to a^+} f(x) = L$

Ex. Given the following graph evaluate the following quantities:



a) $\lim_{x\to 2^-} f(x)$ b) $\lim_{x\to 2^+} f(x)$

c) $\lim_{x\to 2} f(x)$ d) $f(2)$

e) $\lim_{x\to 7^-} f(x)$ f) $\lim_{x\to 7^+} f(x)$

g) $\lim_{x\to 7} f(x)$ h) $f(7)$

Ex. Sketch a graph of an example of a function *f*(*x*) that satisfies the following conditions:

$\lim_{x\to -1^-} f(x) = 4$ $\lim_{x\to -1^+} f(x) = -2$ $f(-1)$ is undefined

$\lim_{x\to 3} f(x) = 0$ $f(3) = 6$