MATH 1910
Section 2.3 Calculating Limits Using the Limit Laws

Look at the limit laws beginning on page 111 of the textbook. An important note is that the limit laws can only be applied to functions whose limits EXIST.

Ex. Given that

$$\lim_{x\to a} f(x) = 2 \qquad \lim_{x\to a} g(x) = -4 \qquad \lim_{x\to a} h(x) = 1,$$

evaluate the following limits, if they exist.

a) $\lim_{x\to a}[3f(x) - h(x)]$ b) $\lim_{x\to a}[g(x)]^2$ c) $\lim_{x\to a} \frac{g(x)}{h(x)} + f(x)$

d) $\lim_{x\to a} \frac{1}{f(x)}$ e) $\lim_{x\to a} \sqrt[3]{h(x) - 3g(x)}$

Ex. Evaluate the following limits, if they exist, without the aid of a calculator:

a) $\lim_{x\to -1} \frac{x-2}{x^2+4x-3}$ b) $\lim_{x\to -1} \sqrt{x^3+2x+7}$

There are times when you have to do some algebraic maneuvering such as factoring both sides of a rational expression, rationalizing the NUMERATOR of a rational expression **before** you can evaluate a limit.

Ex. Evaluate the following limits, if they exist, without the aid of a calculator:

a) $\lim_{x\to -3} \frac{x^2 - x - 12}{x+3}$

$= \lim_{x\to -3} \frac{(x-4)(x+3)}{x+3}$

cancel out the $(x+3)$

$= \lim_{x\to -3} (x-4) = -7$

b) $\lim_{x\to 1} \frac{x^2-1}{x^3-1}$

$= \lim_{x\to 1} \frac{(x+1)(x-1)}{(x-1)(x^2+x+1)}$

$= \lim_{x\to 1} \frac{(x+1)}{(x^2+x+1)}$

$= \frac{2}{3}$

c) $\lim_{h\to 0} \frac{\sqrt{5+h} - \sqrt{5}}{h}$

$= \lim_{h\to 0} \frac{\sqrt{5+h} - \sqrt{5}}{h} \cdot \frac{\sqrt{5+h} + \sqrt{5}}{\sqrt{5+h} + \sqrt{5}}$

rationalize the numerator

$= \lim_{h\to 0} \frac{5+h-5}{h\cdot(\sqrt{5+h} + \sqrt{5})}$

$= \lim_{h\to 0} \frac{h}{h\cdot(\sqrt{5+h} + \sqrt{5})}$

$= \lim_{h\to 0} \frac{1}{\sqrt{5+h} + \sqrt{5}} = \frac{1}{2\sqrt{5}}$

Ex. Evaluate the one sided limit $\lim_{x\to 0^+} \left[\frac{1}{x} - \frac{1}{|x|}\right]$ using your knowledge of the absolute value function.

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Ex. For the function $f(x) = \begin{cases} e^x, & x < 0 \\ 1 - 3x, & 0 < x \le 2 \\ \frac{1}{4}x^2, & x > 2 \end{cases}$. Which of the following limits exist:

a) $\lim\limits_{x \to 0} f(x)$ b) $\lim\limits_{x \to 1} f(x)$ c) $\lim\limits_{x \to 2} f(x)$

HINT: Try **not** to graph the function, just use the y-values on the different intervals and your knowledge of the functions involved.

THE SQUEEZING THEOREM: If $f(x) \le g(x) \le h(x)$ when x is near a (except possible at a) and $\lim\limits_{x \to a} f(x) = \lim\limits_{x \to a} h(x) = L$ then $\lim\limits_{x \to a} g(x) = L$

Ex. Use the Squeeze Theorem to show that $\lim\limits_{x \to 0} 3x \sin\left(\frac{1}{x}\right) = 0$.

First, notice we can't use the limit laws and split the limit up as $\lim\limits_{x \to 0} 3x \cdot \lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right)$ because $\lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist. But, we do know one thing about the values of $\sin\left(\frac{1}{x}\right)$, due to the nature of the trig function,

$$-1 \le \sin\left(\frac{1}{x}\right) \le 1.$$

How can we now apply the squeeze theorem?