DIFFERENTIATION RULES

3.8 LINEAR APPROXIMATIONS AND DIFFERENTIALS

I. $\frac{dy}{dx} = f(x)$ can be interpreted to mean that the derivative is the ratio of the change in y and the change in x

II. When dy and dx are treated as variables, they are called differentials

- A. The differential dx is the independent variable
- B. The differential dy is the dependent variable
- C. $dy = f(x)dx$ shows that the change in y is dependent on the change in x

III. For a very small change in x, the differential dx is very close to the actual change Δx

IV. Example 3 on p. 257

- A. $V = \frac{4}{3}\pi r^3$, so $dV = 4\pi r^2 dr$
- B. r is measured as 21 cm with a maximum possible error of .05 cm
- C. The maximum error in the calculated volume is $dV = 4\pi(21)^2(.05) = 88.2\pi \approx 277 cm^3$
- D. The relative error is a better measure of the error: $\frac{\Delta V}{V} \approx \frac{dV}{V} = \frac{4\pi r^2 dr}{\frac{4}{3}\pi r^3} = 3\frac{dr}{r}$
  1. The relative error in the volume is about three times the relative error in the radius
  2. $3\frac{dr}{r} = 3\frac{.05}{21} \approx .007$ relative error in the volume
- E. There is a percentage error of .7% in the volume

V. Exercise 17 on p. 258

- A. Must have consistent units!
- B. d = 50 m, so r = 25 m; $\Delta r = .05 \text{ cm} = .0005 \text{ m}$
- C. $V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right) = \frac{2}{3}\pi r^3$, so $dV = 2\pi r^2 dr$
- D. $dV = 2\pi(25)^2(.0005) = .625\pi \approx 1.96 m^3$ of paint
  1. If $1 \text{ ft}^3 \approx 7.48 \text{gal}$, how many gallons of paint are needed?
  2. $1.96 \text{ m}^3 \bullet \frac{(100\text{cm})^3}{\text{m}^3} \bullet \frac{(1'')^3}{(2.54\text{cm})^3} \bullet \frac{(1')^3}{(12'')^3} \bullet \frac{7.48\text{gal}}{\text{ft}^3} \approx 517.74\text{gal}$