APPLICATIONS OF DIFFERENTIATION

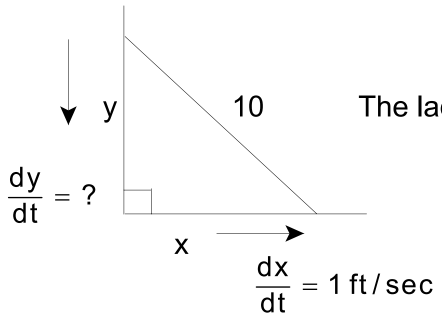
4.1 RELATED RATES

I. Compute the rate of change of one quantity in terms of the rate of change of another quantity.

II. Normally differentiate all quantities with respect to ***time.***

III. Example 2 on p. 266

A. Make a diagram:



The ladder is ***always*** 10 ft long

The change in x is positive because x is increasing.

B. Label diagram: variables, rates of change (derivatives), numerical values which do ***not change***

C. Write an equation which relates all the variables: $x^2 + y^2 = 10^2$

D. Differentiate implicitly with respect to ***time***: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \Rightarrow x\frac{dx}{dt} + y\frac{dy}{dt} = 0$

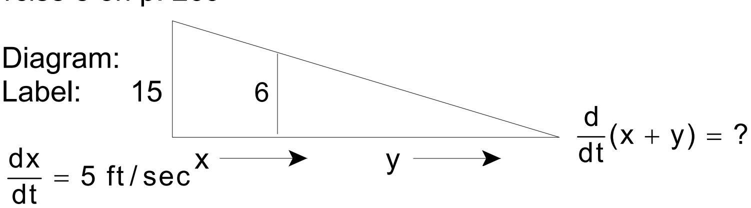
E. Evaluate "when": When x = 6, we can used the Pythagorean Theorem to get y = 8.

F. $x\frac{dx}{dt} + y\frac{dy}{dt} = 0 \Rightarrow 6(1) + 8\frac{dy}{dt} = 0 \Rightarrow \frac{dy}{dt} = -\frac{3}{4}$

G. Answer question(s) completely, include units and "when": The top of the ladder is sliding down (because rate of change of y is negative) the wall at a rate of $\frac{3}{4}$ ft per second when the foot of the ladder is 6 ft from the wall and the foot is sliding away from the wall at a rate of 1 ft/second.

IV. Exercise 8 on p. 269

A. Diagram:
B. Label:



C. Equation: Based on similar triangles: $\frac{15}{6} = \frac{x+y}{y} \Rightarrow 6x + 6y = 15y \Rightarrow y = \frac{2}{3}x$

D. Differentiate implicitly with respect to time: $\frac{d}{dt}(x+y) = \frac{d}{dt}\left(x + \frac{2}{3}x\right) = \frac{d}{dt}\left(\frac{5}{3}x\right) = \frac{5}{3}\frac{dx}{dt}$

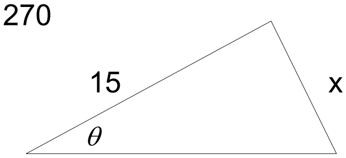
E. Evaluate "when": When x = 40: this is actually irrelevant because there is no x in our differentiated equation!

F. $\frac{5}{3}\frac{dx}{dt} = \frac{5}{3}(5) = \frac{25}{3}$

G. Answer: The tip of his shadow is always moving at a rate of $8\frac{1}{3}$ ft per second whenever the man is walking away from the light pole at a rate of 5 ft per second.

V. Exercise 24 on p. 270

A. Diagram:



$\frac{d\theta}{dt} = 2^{o} / \text{min}$

C. Equation: Use Law of Cosines: $x^2 = 12^2 + 15^2 - 2(12)(15)\cos\theta$

D. Differentiate implicitly: θ $2x\frac{dx}{dt} = -360(-\sin\theta)\frac{d\theta}{dt} \Rightarrow x\frac{dx}{dt} = 180\sin\theta\frac{d\theta}{dt}$

E. Evaluate "when": $\theta = 60^{o} \Rightarrow x^2 = 369 - 360\left(-\frac{1}{2}\right) = 189 \Rightarrow x = \sqrt{189} = 3\sqrt{21}$

F. $x\frac{dx}{dt}180\sin\theta\frac{d\theta}{dt} \Rightarrow 3\sqrt{21}\frac{dx}{dt} = 180\sin 60^{o}\frac{\pi}{90} \Rightarrow \frac{dx}{dt} = \frac{\sqrt{3}\pi}{3\sqrt{21}} = \frac{\pi}{3}\sqrt{\frac{1}{7}} \approx .396$ Must use ***radians***!

G. Answer: The length of the third side is increasing by approximately .396 square meters per minute when the angle between the given sides has measure 60^{o} and is increasing at a rate of 2^{o} per minute.