MATH 1910
Quiz 1 ANSWERS: Tangent Lines and Velocity

For the function $y = \frac{4}{x} + \frac{x}{4}$ …

1. Calculate the slope of the secant line (the average rate of change) between the values of *x* below.

a) between $x = 1$ and $x = 2$ SOLUTION: $m = \frac{f(2) - f(1)}{2 - 1} = \frac{2.5 - 4.25}{1} = -1.75$

b) between $x = 1.5$ and $x = 2$ SOLUTION: $m = \frac{f(2) - f(1.5)}{2 - 1.5} = -\frac{13}{12} \approx -1.083$

c) between $x = 1.9$ and $x = 2$ SOLUTION: $m = \frac{f(2) - f(1.9)}{2 - 1.9} = -\frac{61}{760} \approx -.8026$

d) between $x = 1.999$ and $x = 2$ SOLUTION: $m = \frac{f(2) - f(1.999)}{2 - 1.999} \approx -.7505$

What value do the secant slopes approach as the *x* values get closer to *x* = 2? SOLUTION: These slopes are approaching –¾

Use this slope to get the equation of the tangent line to $y = \frac{4}{x} + \frac{x}{4}$ at $x = 2$.

SOLUTION: using a slope of *m* = –¾ and a point value of (2, 2.5) the line will be $y = -\frac{3}{4}x + 4$

2. An object is thrown upward at an initial velocity of 19.4 meters per second and the height of the object as a function of time is given by the equation $H(t) = 1.8 + 19.4t - 4.9t^2$. (This function models vertical motion only.)

Calculate the average velocity on the following time intervals:

a) $t = 3$ sec. and $t = 4$ sec. SOLUTION: $V_{avg} = \frac{H(4) - H(3)}{4 - 3} \approx -14.9$ meters per second

b) $t = 3$ sec. and $t = 3.5$ sec. SOLUTION: $V_{avg} = \frac{H(3.5) - H(3)}{3.5 - 3} \approx -12.45$ meters per second

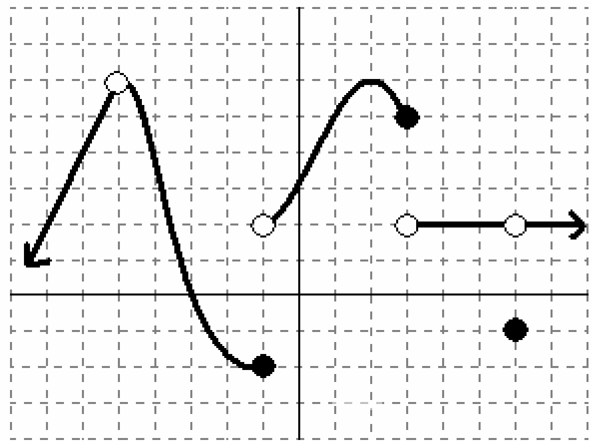
c) $t = 3$ sec. and $t = 3.1$ sec. SOLUTION: $V_{avg} = \frac{H(3.1) - H(3)}{3.1 - 3} \approx -10.49$ meters per second

d) $t = 3$ sec. and $t = 3.001$ sec. SOLUTION: $V_{avg} = \frac{H(3.001) - H(3)}{3.001 - 3} \approx -10.005$ meters per second

What do you estimate to be the instantaneous velocity of the object at $t = 3$ seconds?
SOLUTION: As the time interval surrounding *t* = 3 seconds gets smaller and smaller, the average velocity over the time intervals gets closer and closer to –10 meters per second.

The significance of the negative sign is that the object is moving down at that particular time.

Use the graph of the function $f(x)$ to state the value of each limit, if it exists. If it does not exist, explain why.



a) $f(-5) =$ is undefined

b) $\lim_{x\to -5^-} f(x) = 6$

c) $\lim_{x\to -5^+} f(x) = 6$

d) $\lim_{x\to -5} f(x) = 6$ (same value from both directions)

e) $\lim_{x\to 3^-} f(x) = 5$

f) $\lim_{x\to 3^+} f(x) = 2$

g) $\lim_{x\to 3} f(x) =$ does not exists because the left hand approach does not equal the right hand approach.

h) $f(-1) = -2$ i) $\lim_{x\to -1} f(x) =$ does not exist, left hand limit $\neq$ right hand limit

j) $f(6) = -2$ k) $\lim_{x\to 6} f(x) = 2$

2. Guess the value of the limits below by using either a table or a graph. Make sure to check the limit value from BOTH directions!

a) $\lim_{x\to 0} \dfrac{1 - x - \cos 2x}{x}$

The graph approaches a y-value of –1 as x approaches 0 from both sides of $x = 0$.
This limit's value is –1

b) $\lim_{x\to 2} \dfrac{|x-2|}{2-x}$

The graph here does NOT approach the same y-value from both sides.
The left hand approach is 1 and the right hand approach is –1. This limit does not exist.

3. Construct a function which has the following attributes:

$\lim_{x\to -1} f(x) = -2$, $f(-1) = 4$,
$\lim_{x\to 2^-} f(x) = 1$, $\lim_{x\to 2^+} f(x) = -1$, $f(2)$ is undefined

(answers may vary)