3.1 HW

3. $f(x) = x^2 + 9x$, $a = 0$

① $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \rightarrow f'(0) = \lim_{h\to 0} \frac{(0+h)^2 + 9(0+h) - (0^2 + 9(0))}{h} = \lim_{h\to 0} \frac{h^2+9h}{h}$

$f'(0) = \lim_{h\to 0} \frac{h(h+9)}{h} = \lim_{h\to 0} h+9 = \boxed{9}$

② $f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} \rightarrow f'(0) = \lim_{x\to 0} \frac{x^2+9x-0}{x-0} = \lim_{x\to 0} \frac{x(x+9)}{x}$

$f'(0) = \lim_{x\to 0} x+9 = \boxed{9}$

5. $f(x) = 3x^2 + 4x + 2$, $a = -1$

① $f'(a) = \lim_{h\to 0} \frac{f(a+h) - f(a)}{h} \rightarrow f'(-1) = \lim_{h\to 0} \frac{[3(-1+h)^2 + 4(-1+h) + 2] - [3(-1)^2 + 4(-1) + 2]}{h}$

$f'(-1) = \lim_{h\to 0} \frac{[\cancel{3} - 6h + 3h^2 - \cancel{4} + 4h + \cancel{2}] - [\cancel{1}]}{h} = \lim_{h\to 0} \frac{h(-6+3h+4)}{h}$

$f'(-1) = \lim_{h\to 0} \frac{-2+3h}{1} = \boxed{-2}$

② $f'(a) = \lim_{x\to a} \frac{f(x) - f(a)}{x-a} \Rightarrow f'(-1) = \lim_{x\to -1} \frac{3x^2+4x+2 - [3(-1)^2 + 4(-1) + 2]}{x+1}$

$f'(-1) = \lim_{x\to -1} \frac{3x^2+4x+1}{x+1} = \lim_{x\to -1} \frac{(x+1)(3x+1)}{x+1} = \lim_{x\to -1} (3x+1) = \boxed{-2}$

7. $m_{sec}[2, 2.5] = \frac{f(2.5) - f(2)}{2.5-2} = \frac{2.5-2}{2.5-2} = 1$

let red line $= h(x)$

$f'(2) = m_{tan} = \frac{h(3) - h(2)}{3-2} = \frac{2.8-2}{1} = .8$

slope steepens after $x = 2$ so secant line has greater slope.

9. $f'(1) \approx 0$, $f'(2) \approx .8$

11. $f'(1) = 0$, $f'(2) = 0$, $f'(4) = 1$, $f'(7) = 0$

13. $f'(5.5) > f'(6.5)$ because graph is decreasing in slope during that region.

15. $f(x) = 7x-9$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} \to \lim_{x\to a} \frac{(7x-9)-(7a-9)}{x-a} = \lim_{x\to a} \frac{7(x-a)}{x-a} = \boxed{7}$$

17. $g(t) = 8-3t$

$$\lim_{h\to 0} \frac{g(t+h)-g(t)}{h} = \lim_{h\to 0} \frac{(8-3(t+h)) - (8-3t)}{h} = \lim_{h\to 0} \frac{(8-3t-3h)-(8-3t)}{h}$$

$$= \lim_{h\to 0} \frac{-3h}{h} = \boxed{-3}$$

19. $f'(3) = 2 \to m = 2$, $f(3) = 5 \to (3,5)$

$y-5 = 2(x-3)$

$\boxed{y = 2x-1}$

29. $f(t) = t-2t^2$, $a = 3$

$$\lim_{t\to a} \frac{f(t)-f(a)}{t-a} = \lim_{t\to 3} \frac{t-2t^2-(3-2(3)^2)}{t-3} = \lim_{t\to 3} \frac{-2t^2+t+15}{t-3}$$

$$= \lim_{t\to 3} \frac{(t-3)(-2t-5)}{t-3} = \lim_{t\to 3} (-2t-5) = -2(3)-5 = \boxed{-11}$$

31. $f(x) = x^3+x$, $a = 0$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to 0} \frac{x^3+x-0}{x-0} = \lim_{x\to 0} \frac{x(x^2+1)}{x} = \lim_{x\to 0} (x^2+1) = \boxed{1}$$

33. $f(x) = \frac{1}{x}$, $a = 8$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to 8} \frac{\frac{1}{x}-\frac{1}{8}}{x-8} \cdot \frac{8x}{8x} = \lim_{x\to 8} \frac{8-x}{(x-8)(8x)}$$

$$= \lim_{x\to 8} \frac{-1\cancel{(8+x)}}{\cancel{(x-8)}(8x)} = \lim_{x\to 8} \frac{-1}{8x} = \boxed{\frac{-1}{64}}$$

35. $f(x) = \frac{1}{x+3}$, $a=-2$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to -2} \frac{\frac{1}{x+3} - \frac{1}{-2+3}}{x+2} = \lim_{x\to -2} \frac{\left(\frac{1}{x+3} - 1\right)}{(x+2)} \frac{(x+3)}{(x+3)}$$

$$= \lim_{x\to -2} \frac{1-(x+3)}{(x+2)(x+3)} = \lim_{x\to -2} \frac{-x-2}{(x+2)(x+3)} = \lim_{x\to -2} \frac{-1(x+2)}{(x+2)(x+3)}$$

$$= \lim_{x\to -2} \frac{-1}{x+3} = \frac{-1}{-2+3} = \boxed{-1}$$

tan. line: $y-1=-1(x+2)$

$y=-1x-1$

37. $f(x) = \sqrt{x+4}$, $a=1$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to 1} \frac{\sqrt{x+4} - \sqrt{1+4}}{x-1} = \lim_{x\to 1} \frac{(\sqrt{x+4}-\sqrt{5})(\sqrt{x+4}+\sqrt{5})}{(x-1)(\sqrt{x+4}+\sqrt{5})}$$

$$= \lim_{x\to 1} \frac{x+4-5}{(x-1)(\sqrt{x+4}+\sqrt{5})} = \lim_{x\to 1} \frac{x-1}{(x-1)(\sqrt{x+4}+\sqrt{5})} = \lim_{x\to 1} \frac{1}{\sqrt{x+4}+\sqrt{5}} = \frac{1}{2\sqrt{5}}$$

tan. line → $y-\sqrt{5} = \frac{1}{2\sqrt{5}}(x-1)$

$y = \frac{1}{2\sqrt{5}}x + \sqrt{5} - \frac{1}{2\sqrt{5}}$

39. $f(x) = \frac{1}{\sqrt{x}}$, $a=4$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to 4} \frac{\frac{1}{\sqrt{x}} - \frac{1}{2}}{x-4} \cdot \frac{2\sqrt{x}}{2\sqrt{x}} = \lim_{x\to 4} \frac{2-\sqrt{x}}{(x-4)\,2\sqrt{x}}$$

$$= \lim_{x\to 4} \frac{2-\sqrt{x}}{(\sqrt{x}-2)(\sqrt{x}+2)\,2\sqrt{x}} = \lim_{x\to 4} \frac{-1(\sqrt{x}-2)}{(\sqrt{x}-2)(\sqrt{x}+2)(2\sqrt{x})} = \lim_{x\to 4} \frac{-1}{(\sqrt{x}+2)\,2\sqrt{x}}$$

$$= \frac{-1}{4(4)} = \frac{-1}{16}$$

tan. line: $y-\frac{1}{2} = \frac{-1}{16}(x-4)$

$y = \frac{-1}{16}x + \frac{3}{4}$

41. $f(t) = \sqrt{t^2+1}$, $a=3$

$$\lim_{t\to a} \frac{f(t)-f(a)}{t-a} = \lim_{t\to 3} \frac{\sqrt{t^2+1} - \sqrt{10}}{t-3} \cdot \frac{\sqrt{t^2+1}+\sqrt{10}}{\sqrt{t^2+1}+\sqrt{10}} = \lim_{t\to 3} \frac{t^2+1-10}{(\sqrt{t^2+1}+\sqrt{10})(t-3)}$$

(t^2-9)

$$= \lim_{t\to 3} \frac{(t-3)(t+3)}{(t-3)(\sqrt{t^2+1}+\sqrt{10})} = \lim_{t\to 3} \frac{t+3}{\sqrt{t^2+1}+\sqrt{10}} = \frac{6}{2\sqrt{10}} = \frac{3}{\sqrt{10}}$$

tan. line

$y-\sqrt{10} = \frac{3}{\sqrt{10}}(x-3)$

$y = \frac{3}{\sqrt{10}}x - \frac{9}{\sqrt{10}} + \sqrt{10}$

$y = \frac{3}{\sqrt{10}}x + \frac{1}{\sqrt{10}}$