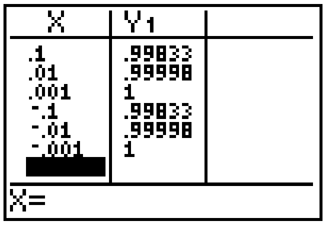
INTRODUCTION TO LIMITS

Ex. What do the y-values of the graph of $f(x)=\dfrac{\sin x}{x}$ approach as the x-values approach 0?

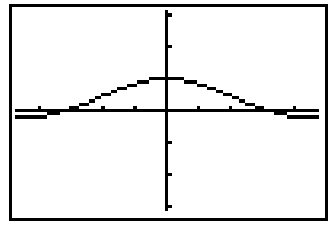
Look at a table of values for the function as x → 0 (use ASK mode)



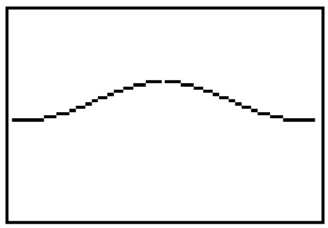
X	Y1	
.1	.99833	
.01	.99998	
.001	1	
-.1	.99833	
-.01	.99998	
-.001	1	

X=

Look at the graph as x → 0 (ZOOMDEC)



Based on the numerical data and the graphical data the graph seems to approach y = 1 as x → 0.
But, the function is not defined at x = 0.
Look at the graph with the axes turned off. (2nd FORMAT, AxesOff)



You can barely tell, but there is a hole in the graph at x = 0. The function is not defined there.

<u>DEFINITION OF LIMIT</u>: We write

$$\lim_{x\to a} f(x)=L \text{ and say "the limit of } f(x) \text{ as } x \text{ approaches } a\text{, equals } L\text{"}$$

if we can make the values of *f(x)* arbitrarily close to *L* (as close to *L* as we like) by taking *x* to be sufficiently close to equal *a*.
When evaluating limits, you're actually finding the "intended value" of the function as it approaches a certain x-value.
In the above example the function $f(x)=(\sin x)/x$ is not defined at x = 0, but we can say that the limit as x approaches 0 does equal 1 because that is the "intended" y-value the graph approachs.

So $\displaystyle\lim_{x\to 0}\frac{\sin x}{x}$ does exist and $\displaystyle\lim_{x\to 0}\frac{\sin x}{x}$ = 1, regardless of whether or not the function is defined there.

Ex. Evaluate these limits either graphically or numerically (using your table):

a) $\displaystyle\lim_{x\to 2} 2x^2-3x+5$ b) $\displaystyle\lim_{x\to -1}\frac{x^2-3x-4}{x+1}$ c) $\displaystyle\lim_{x\to 0}\frac{\sqrt{x+16}-4}{x}$ d) $\displaystyle\lim_{x\to 0}\frac{1}{1-2^{1/x}}$

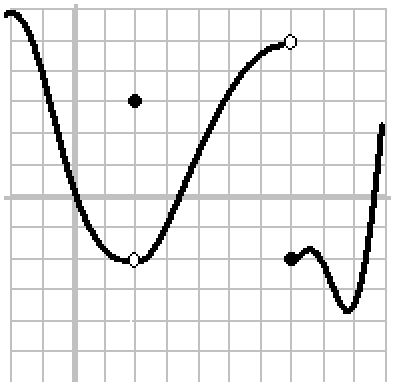
Left hand and Right hand limits: When using your table to evaluate these limits you've considered values to the left and to the right of the central x value you're approaching. These are called the left hand and right hand limits of the function *f (x).*

$\displaystyle\lim_{x\to a^-} f(x) = L$ (taking values of x approaching a from the left, x < a)

$\displaystyle\lim_{x\to a^+} f(x) = L$ (taking values of x approaching a from the right, x > a)

IMPORTANT! → The limit $\displaystyle\lim_{x\to a} f(x)=L$ exists ONLY if $\displaystyle\lim_{x\to a^-} f(x) = L$ AND $\displaystyle\lim_{x\to a^+} f(x) = L$

Ex. Given the following graph evaluate the following quantities:



a) $\lim_{x \to 2^-} f(x)$

b) $\lim_{x \to 2^+} f(x)$

c) $\lim_{x \to 2} f(x)$

d) $f(2)$

e) $\lim_{x \to 7^-} f(x)$

f) $\lim_{x \to 7^+} f(x)$

g) $\lim_{x \to 7} f(x)$

h) $f(7)$

Ex. Sketch a graph of an example of a function $f(x)$ that satisfies the following conditions:

$\lim_{x \to -1^-} f(x) = 4$

$\lim_{x \to -1^+} f(x) = -2$

$f(-1)$ is undefined

$\lim_{x \to 3} f(x) = 0$

$f(3) = 6$