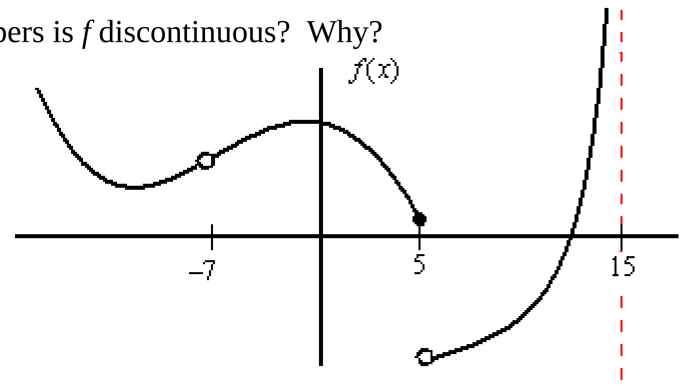
2.4 Limits and Continuity
Definition of Continuity: A function $f(x)$ is **continuous at a number *a*** if $\lim_{x \to a} f(x) = f(a)$.

The formal definition of continuity requires 3 things:

1. The existence of a limit $\lim_{x \to a} f(x)$
2. The function to $f(x)$ must be defined at $x = a$
3. $\lim_{x \to a} f(x) = f(a)$

So not only does the limit exist, but this means as *x* approaches *a* the *y*-values of $f(x)$ approach $f(a)$ AND $f(a)$ is a defined value on the graph of the function. If a function is not continuous at a specific point it is said to be **discontinuous** there.

1. Consider the function graphed here. At what numbers is *f* discontinuous? Why?



One intuitive idea of continuity comes from the function's graph. If you're able to trace the graph of a function without lifting your pen from the paper then the function is continuous.

2. Using their graphs determine where these functions discontinuous?

a) $f(x) = \dfrac{2x^2 - 9x - 5}{2x + 1}$ b) $g(x) = \begin{cases} \frac{1}{4x^3} & x \neq 0 \\ 0 & x = 0 \end{cases}$ c) $h(x) = \dfrac{|x+1|}{x+1}$ d) $y = \dfrac{\sin x}{x}$

In problems a) and d) above, the limits exist at their discontinuous points, but the function is not defined there. These could be remedied by "redefining" the function to include the point on the graph that was punched out. These kinds of discontinuities that can be fixed are called **removable discontinuities**.

In problems b) and c) above the discontinuities occur when the graph either encounters a vertical asymptote or a "jump" point. These are called **infinite** or **jump discontinuities** and can't be removed like the other ones.

3. How could a) and d) be redefined to make them continuous functions?

4. For what values of *c* will the following function be continuous? $f(x) = \begin{cases} cx^2 - 2 & x \leq 1 \\ 2cx + 5 & x > 1 \end{cases}$

Continuity from the Left and Right: A function <u>*f* is continuous from the right at a number *a*</u> if $\lim_{x \to a^+} f(x) = f(a)$ and <u>*f* is continuous from the left at a number *a*</u> if $\lim_{x \to a^-} f(x) = f(a)$.

Continuity on an Interval: A function is continuous on an interval if it is continuous at every number within the interval. (At the endpoints continuity is understood to mean left continuous and right continuous)

Building New Continuous Functions from Old Ones

Given two continuous functions f and g which are continuous at a number a, the following functions are also continuous at the number a:

1. $f+g$ 2. $f-g$ 3. cf where c is some constant
4. fg 5. $\frac{f}{g}$ if $g(a)\neq 0$

5. Explain why the function $g(x)=x^2\cos(3x)$ is continuous at every number in its domain. What is its domain?

Limits and Continuity: When evaluating limits, your first attempt should be to simply "plug in" the x-value you're approaching. When you're able to do this, you're using the continuity of the function to evaluate the limit.

6. Use continuity to evaluate the following limits:

a) $\lim_{x\to 0} 3x^2+2$ b) $\lim_{x\to 3}\frac{2x+7}{x-8}$

SOLUTION: In each case, the value of x you're approaching is within the domain of the function, therefore the function is continuous there. Simply "plug in" the x-value to determine the intended y-value of the limit

a) $\lim_{x\to 0} 3x^2+2=2$ b) $\lim_{x\to 3}\frac{2x+7}{x-8}=-\frac{13}{5}$