10.4 HW

1. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$ converges absolutely since

$\sum_{n=0}^{\infty} \left|\frac{(-1)^n}{2^n}\right| = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n$ is a convergent geometric series $(r<1)$.

3. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}}$

absolutely? $\rightarrow \sum_{n=1}^{\infty} \left|\frac{(-1)^{n-1}}{n^{1/3}}\right| = \sum_{n=1}^{\infty} \frac{1}{n^{1/3}}$ this is a divergent p-series $(p<1)$.

So $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}}$ is <u>NOT</u> absolutely convergent.

$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}}$ converges (conditionally) since $\lim_{n\to\infty} \frac{1}{n^{1/3}} = 0$.

5. $\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{(1.1)^n}$

absolutely? $\rightarrow \sum_{n=0}^{\infty} \left|\frac{(-1)^{n-1}}{(1.1)^n}\right| = \sum_{n=0}^{\infty} \left(\frac{1}{1.1}\right)^n$ This is a convergent geometric series with $r<1$.

So $\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{(1.1)^n}$ converges absolutely.

7. $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$

absolutely? $\rightarrow \sum_{n=2}^{\infty} \left|\frac{(-1)^n}{n\ln n}\right| = \sum_{n=2}^{\infty} \frac{1}{n\ln n}$

Not wrong but not helpful $\left\{ \begin{array}{l} \text{LCT with divergent } \frac{1}{n} \rightarrow L = \lim_{n\to\infty} \frac{\frac{1}{n\ln n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{n\ln n} \\ L = \lim_{n\to\infty} \frac{1}{\ln n} = \frac{1}{\infty} = 0 \rightarrow \text{LCT is inconclusive.} \end{array} \right.$

IT $\rightarrow \frac{1}{n\ln n}$ are values of $f(x) = \frac{1}{x\ln x}$.

$\lim_{b\to\infty} \int_2^b \frac{1}{x\ln x}\,dx$ $\quad u = \ln x$, $du = \frac{1}{x}dx$ $\rightarrow \lim_{b\to\infty} \int_{\ln 2}^{b} \frac{1}{u}\,du$

$= \lim_{b\to\infty} \left[\ln|u|\right]_2^b = \infty$

Since $\lim_{b\to\infty}\int_2^{\infty} \frac{1}{x\ln x}\,dx$ diverges $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$ diverges

and $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$ is NOT absolutely convergent.

However, $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{n\ln n} = 0$ $\therefore$ $\sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n}$

converges conditionally.

9. $\sum_{n=2}^{\infty} \frac{\cos n\pi}{(\ln n)^2} = \sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$ Use LCT with

absolutely? Does $\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$ converge?

divergent $B_n = \frac{1}{n}$

$L = \lim_{n\to\infty} \frac{\frac{1}{(\ln n)^2}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{n}{(\ln n)^2} \overset{H}{=} \lim_{n\to\infty} \frac{1}{2(\ln n)\frac{1}{n}}$

$L = \lim_{n\to\infty} \frac{n}{2\ln n} \overset{H}{=} \lim_{n\to\infty} \frac{1}{2(\frac{1}{n})} = \lim_{n\to\infty} \frac{n}{2} = \infty$

$\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$ diverges so $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$ is <u>NOT</u> absolutely

convergent. $\sum_{n=2}^{\infty} \frac{(-1)^n}{(\ln n)^2}$ does converge conditionally

though since $\lim_{n\to\infty} \frac{1}{(\ln n)^2} = 0$.