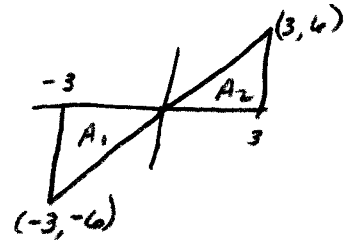
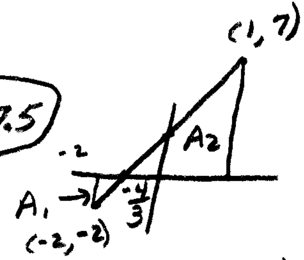
5.2 HW

1. $\int_{-3}^{3} 2x\,dx = 0$



$A_1 + A_2 = 0$

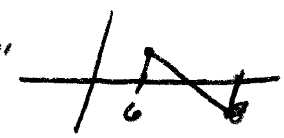
3. $\int_{-2}^{1} (3x+4)\,dx = A_1 + A_2 = \frac{45}{6} = \frac{15}{2} = 7.5$



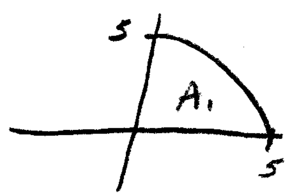
$3x+4=0$
$3x=-4$
$x=\frac{-4}{3}$

$A_1 = -\frac{1}{2}\left(\frac{2}{3}\right)(2) = \frac{-2}{3}$
$A_2 = \frac{1}{2}\left(\frac{7}{3}\right)(7) = \frac{49}{6}$

5. $\int_{6}^{8} (7-x)\,dx = 0 \leftarrow$ "same area above and below x-axis"

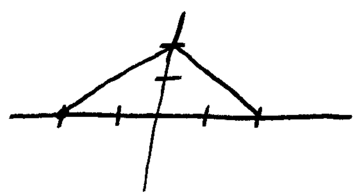


7. $\int_{0}^{5} \sqrt{25-x^2}\,dx = \frac{25\pi}{4}$



$\frac{1}{4}$th of circle radius 5.
$A_1 = \frac{1}{4}\pi(5)^2 = \frac{25\pi}{4}$

9. $\int_{-2}^{2} (2-|x|)\,dx = 4$



$A = \frac{1}{2}(4)(2) = 4$

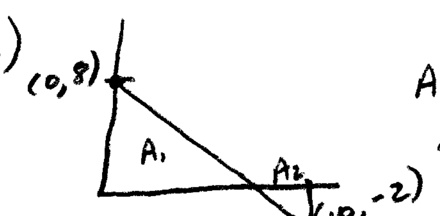
11. $\int_{0}^{10} (8-x)\,dx$

a.) $R_n = \lim_{n\to\infty} \sum_{i=1}^{n} f(a + i\Delta x)\Delta x = \lim_{n\to\infty} \sum f\left(\frac{10i}{n}\right)\left(\frac{10}{n}\right)$

$= \lim_{n\to\infty} \frac{10}{n} \sum \left(8 - \frac{10i}{n}\right) = \lim_{n\to\infty} \frac{80}{n}\sum 1 - \frac{100}{n^2}\sum i$

$= \lim_{n\to\infty} \frac{80}{n}(n) - \frac{100}{n^2}\left(\frac{n(n+1)}{2}\right) = 80 - 50 = 30$

b.)



$A_1 = \frac{1}{2}(8)(8) = 32$
$A_2 = -\frac{1}{2}(2)(2) = -2$

$A_1 + A_2 = 30$

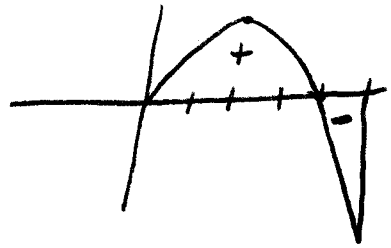
13. a.) $\int_0^2 f(x)dx = -\frac{1}{2}\pi(1)^2 = \boxed{\frac{-\pi}{2}}$ b.) $\int_0^6 f(x)dx = -\frac{\pi}{2} + \frac{1}{2}\pi(2)^2 = \frac{3\pi}{2}$

15. $\int_0^3 g(t)dt = -\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) = \boxed{\frac{3}{2}}$

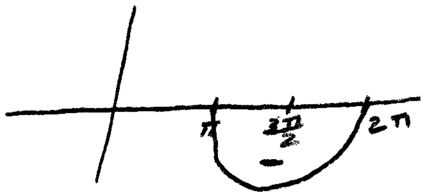
$\int_3^5 g(t)dt = \frac{1}{2}(2)(1) + (-\frac{1}{2})(1)(-2) = \boxed{0}$

23. $\int_0^5 (4x - x^2)dx$

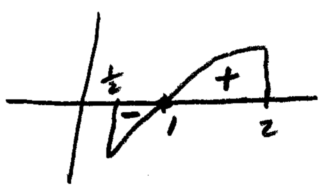
$4x - x^2 = 0$
$x(4-x) = 0$
$x = 0 \quad x = 4$



25. $\int_\pi^{2\pi} \sin x\, dx$



27. $\int_{\frac{1}{2}}^{2} \ln x\, dx$



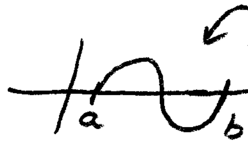
59. $\int_0^3 f(x)dx + \int_3^7 f(x)dx = \boxed{\int_0^7 f(x)dx}$

61. $\int_2^9 f(x)dx - \int_2^5 f(x)dx = \boxed{\int_5^9 f(x)dx}$

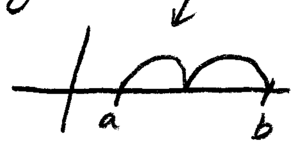
67. $\int_a^b f(x)dx$ is the <u>signed area</u> between $f(x)$ and the x-axis. Areas below are negative. Areas above are positive.

$\int_a^b |f(x)|dx$ is the <u>total area</u> between $f(x)$ and the x-axis. Any area is positive regardless whether original $f(x)$ is above or below x-axis.

$\int f(x)\,dx = 0$



$\int |f(x)|dx \neq 0$



71. $\int_0^6 |3-x|\,dx$ $\quad 3-x=0,\ 3=x \rightarrow |3-x| = \begin{cases} 3-x, & x \le 3 \\ -(3-x), & x > 3 \end{cases}$

$$= \int_0^3 (3-x)\,dx + \int_3^6 -(3-x)\,dx$$

$$= \int_0^3 (3-x)\,dx - \int_3^6 (3-x)\,dx$$

$$= \left[3x - \frac{x^2}{2}\right]_0^3 - \left[3x - \frac{x^2}{2}\right]_3^6 = \left(9-\frac{9}{2}\right) - (0) - (0) + \left(9 - \frac{9}{2}\right) = 9$$

73. $|x^3| = \begin{cases} -x^3, & x < 0 \\ x^3, & x \ge 0 \end{cases}$ so... $\int_{-1}^1 x^3\,dx = -\int_{-1}^0 x^3\,dx + \int_0^1 x^3\,dx$

$$= \left[\frac{x^4}{4}\right]_{-1}^0 + \left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

75. Since $x^5 < x^4$ on $x = (0,1)$, we know $\int_0^1 x^5\,dx < \int_0^1 x^4\,dx$.

Since $x^5 > x^4$ on $x = (1,2)$, we know $\int_1^2 x^5\,dx > \int_1^2 x^4\,dx$.

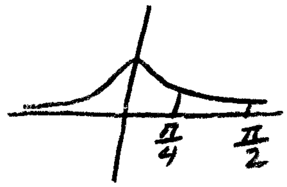
77. We know $y = \sin x$ increases between .2 and .3 radians (angle is in Q I)

so we can say $\int_{.2}^{.3} \sin x\,dx < \sin(.3)(.1) \approx .02955$ upper bound

and $\int_{.2}^{.3} \sin x\,dx > \sin(.2)(.1) \approx .01987$ lower bound

This gives a relatively small range for $\int_{.2}^{.3} \sin x\,dx$. This practice can be useful for estimating integrals of functions that are difficult (or impossible) to integrate.

79. $y = \frac{\sin x}{x}$



since $\frac{\sin x}{x}$ decreases on $x = (\frac{\pi}{4}, \frac{\pi}{2})$

$$L_1 > \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x}{x}\, dx > R_1$$

$$L_1 = \frac{\sin(\frac{\pi}{4})}{\frac{\pi}{4}} \left(\frac{\pi}{4}\right) = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

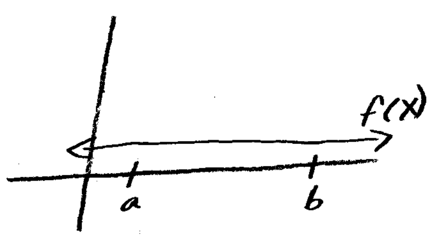
$$R_1 = \frac{\sin(\frac{\pi}{2})}{\frac{\pi}{2}} \left(\frac{\pi}{4}\right) = \frac{\sin(\frac{\pi}{2})}{2} = \frac{1}{2}$$

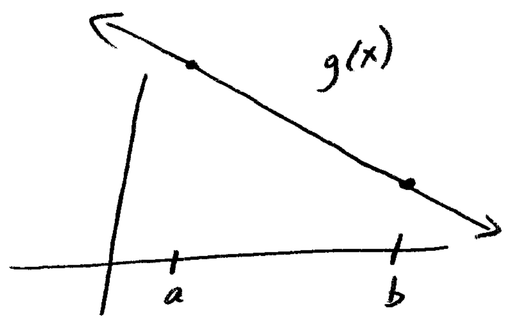
so $\quad \frac{\sqrt{2}}{2} > \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin x}{x}\, dx > \frac{1}{2}$

81. If $\int_a^b f(x)\,dx \le \int_a^b g(x)\,dx$, then $f(x) \le g(x)$ on $[a, b]$.

But $f'(x)$ <u>does not</u> need to be less than $g'(x)$.

For example...





On $[a,b]$ clearly $g(x) > f(x)$ and $\int_a^b g(x)\,dx > \int_a^b f(x)\,dx$.

However, $f'(x) = 0$ and $g'(x) < 0$ thus $f'(x) > g'(x)$.