5.7 HW

1. $u = x^3 - x^2 \rightarrow \frac{du}{dx} = 3x^2 - 2x \rightarrow du = (3x^2 - 2x)dx$

3. $u = \cos(x^2) \rightarrow \frac{du}{dx} = -2x\sin(x^2) \rightarrow du = -2x\sin(x^2)dx$

5. $u = e^{4x+1} \rightarrow \frac{du}{dx} = 4e^{4x+1} \rightarrow du = 4e^{4x+1}dx$

7. $\int (x-7)^3 dx \qquad u = x-7, \ du = dx$

$= \int u^3 du = \frac{u^4}{4} + C = \frac{1}{4}(x-7)^4 + C$

9. $\int (3t-4)^5 dt \qquad u = 3t-4, \ du = 3dt \rightarrow \frac{1}{3}du = dt$

$= \int (u)^5 (\frac{1}{3}du) = \frac{1}{3}\int u^5 du = \frac{1}{3}\left[\frac{u^6}{6} + C\right] = \frac{1}{18}(3t-4)^6 + C$

11. $\int t\sqrt{t^2+1}\,dt \qquad u = t^2+1, \ du = 2t$

$= \frac{1}{2}\int 2t\sqrt{t^2+1}\,dt = \frac{1}{2}\int \sqrt{u}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2} + C\right]$

$= \frac{1}{3}\sqrt{u^3} + C = \frac{1}{3}\sqrt{(t^2+1)^3} + C$

13. $\int \frac{t^3}{(4-2t^4)^{11}}dt, \qquad u = 4-2t^4, \ du = -8t^3 dt$

$= \frac{1}{-8}\int \frac{-8t^3 dt}{(4-2t^4)^{11}} = \frac{-1}{8}\int \frac{du}{u^{11}} = \frac{-1}{8}\left[\frac{1}{-10u^{10}} + C\right]$

$= \frac{1}{80u^{10}} + C = \frac{1}{80(4-2t^4)^{10}} + C$

15. $\int x(x+1)^9 dx \qquad u = x+1, \ du = dx \qquad$ * we know $x = u-1$ too!

$= \int (u-1)u^9 du = \int (u^{10} - u^9)du = \frac{u^{11}}{11} - \frac{u^{10}}{10} + C$

$= \frac{(x+1)^{11}}{11} - \frac{(x+1)^{10}}{10} + C$

17. $\int x^2\sqrt{x+1}\,dx$ $\quad u = x+1,\quad du = dx$

$x = u-1 \rightarrow x^2 = (u-1)^2 = u^2 - 2u + 1$

$= \int (u^2-2u+1)\sqrt{u}\,du = \int (u^{5/2} - 2u^{3/2} + u^{1/2})\,du$

$= \frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2} + C = \frac{2\sqrt{(x+1)^7}}{7} - \frac{4\sqrt{(x+1)^5}}{5} + \frac{2\sqrt{(x+1)^3}}{3} + C$

19. $\int \sin^2\theta\cos\theta\,d\theta$ $\quad u = \sin\theta,\quad du = \cos\theta\,d\theta$

$= \int u^2\,du = \frac{u^3}{3} + C = \frac{\sin^3\theta}{3} + C$

21. $\int x e^{-x^2}\,dx$ $\quad u = -x^2,\quad du = -2x\,dx$

$= \frac{1}{-2}\int -2x e^{-x^2}\,dx = \frac{-1}{2}\int e^u\,du = \frac{-1}{2}[e^u + C] = \frac{-1}{2}e^{-x^2} + C$

23. $\int \frac{(\ln x)^2\,dx}{x}$ $\quad u = \ln x,\quad du = \frac{1}{x}\,dx$

$= \int u^2\,du = \frac{u^3}{3} + C = \frac{1}{3}(\ln x)^3 + C$

25. $\int x^3\cos(x^4)\,dx$ $\quad u = x^4,\quad du = 4x^3\,dx$

$= \frac{1}{4}\int 4x^3\cos(x^4)\,dx = \frac{1}{4}\int \cos u\,du = \frac{1}{4}[\sin u + C]$

$= \frac{1}{4}\sin x^4 + C$

27. $\int x^{1/2}\cos(x^{3/2})\,dx$ $\quad u = x^{3/2},\quad du = \frac{3}{2}x^{\frac{1}{2}}\,dx$

$= \frac{2}{3}\int \frac{3}{2}x^{\frac{1}{2}}\cos(x^{3/2})\,dx = \frac{2}{3}\int \cos u\,du = \frac{2}{3}\sin u + C$

$= \frac{2}{3}\sin(x^{3/2}) + C$

29. $\int (4x+5)^9 dx$ $\qquad u = 4x+9, \quad du = 4dx$

$= \frac{1}{4}\int (4x+5)^9 \cdot 4dx = \frac{1}{4}\int u^9 du = \frac{u^{10}}{40} + C = \boxed{\frac{(4x+9)^{10}}{40} + C}$

31. $\int \frac{dt}{\sqrt{t+12}}$ $\qquad u = t+12, du = dt$

$= \int u^{-1/2} du = 2u^{\frac{1}{2}} + C = 2\sqrt{t+12} + C$

33. $\int \frac{x+1}{(x^2+2x)^3} dx$ $\qquad u = x^2+2x, \quad du = (2x+2)dx = 2(x+1)dx$

$= \frac{1}{2}\int \frac{2(x+1)dx}{(x^2+2x)^3} = \frac{1}{2}\int u^{-3} du = \frac{1}{2}\left[\frac{u^{-2}}{-2} + C\right]$

$= \boxed{\frac{-1}{4(x^2+2x)^2} + C}$

35. $\int \frac{x}{\sqrt{x^2+9}} dx$ $\qquad u = x^2+9, \quad du = 2x\, dx$

$= \frac{1}{2}\int \frac{2x\,dx}{\sqrt{x^2+9}} = \frac{1}{2}\int u^{-\frac{1}{2}} du = \frac{1}{2}\left[2u^{\frac{1}{2}} + C\right]$

$= \boxed{\sqrt{x^2+9} + C}$

37. $\int (3x^2+1)(x^3+x)^2 dx$ $\qquad u = x^3+x, \quad du = (3x^2+1)dx$

$= \int u^2 du = \frac{u^3}{3} + C = \boxed{\frac{(x^3+x)^3}{3} + C}$

39. $\int (3x+8)^{11} dx$ $\qquad u = 3x+8, \quad du = 3dx$

$= \frac{1}{3}\int (3x+8)^{11} \cdot 3dx = \frac{1}{3}\int u^{11} du = \frac{u^{12}}{36} + C = \boxed{\frac{(3x+8)^{12}}{36} + C}$

41. $\int x^2\sqrt{x^3+1}\,dx \qquad u=x^3+1,\ du=3x^2dx$

$= \frac{1}{3}\int 3x^2\sqrt{x^3+1}\,dx = \frac{1}{3}\int u^{\frac{1}{2}}du = \frac{1}{3}\left[\frac{2}{3}u^{3/2}+C\right]$

$= \boxed{\frac{2\sqrt{(x^3+1)^3}}{9}+C}$

43. $\int \frac{dx}{(x+5)^3} \qquad u=x+5,\ du=dx$

$= \int u^{-3}du = \frac{u^{-2}}{-2}+C = \boxed{\frac{-1}{2(x+5)^2}+C}$

45. $\int z^2(z^3+1)^{12}dz \qquad u=z^3+1,\ du=3z^2dz$

$= \frac{1}{3}\int 3z^2(z^3+1)^{12}dz = \frac{1}{3}\int u^{12}du = \frac{u^{13}}{39}+C$

$= \boxed{\frac{(z^3+1)^{13}}{39}+C}$

47. $\int (x+2)(x+1)^{\frac{1}{4}}dx \qquad u=x+1,\ du=dx$

$x=u-1 \rightarrow x+2=u+1$

$= \int (u+1)\,u^{\frac{1}{4}}du = \int\left(u^{5/4}+u^{\frac{1}{4}}\right)du = \frac{4}{9}u^{\frac{9}{4}}+\frac{4}{5}u^{\frac{5}{4}}+C$

$= \boxed{\frac{4\sqrt[4]{(x+1)^9}}{9}+\frac{4\sqrt[4]{(x+1)^5}}{5}+C}$

49. $\int \sin(8-3\theta)\,d\theta \qquad u=8-3\theta,\ du=-3d\theta$

$\frac{1}{-3}\int \sin(8-3\theta)(-3d\theta) = -\frac{1}{3}\int \sin u\,du = -\frac{1}{3}\left[-\cos u+C\right]$

$= \boxed{\frac{1}{3}\cos(8-3\theta)+C}$

51. $\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt$ $\qquad u=\sqrt{t}=t^{\frac{1}{2}},\quad du=\frac{1}{2}t^{-\frac{1}{2}}dt=\frac{1}{2}\cdot\frac{dt}{\sqrt{t}}$

$= 2\int \frac{\cos\sqrt{t}}{2\sqrt{t}}\,dt = 2\int \cos u\,du = 2[\sin u + C]$

$= 2\sin\sqrt{t} + C$

53. $\int \tan(4\theta+9)\,d\theta$ $\qquad u=4\theta+9,\quad du=4d\theta$

$= \frac{1}{4}\int \tan(4\theta+9)(4d\theta) = \frac{1}{4}\int \tan u\,du = \frac{1}{4}\int \frac{\sin u}{\cos u}\,du$

$w=\cos u,\quad dw=-\sin u\,du$

$= -\frac{1}{4}\int \frac{-\sin u}{\cos u}\,du = -\frac{1}{4}\int \frac{1}{w}\,dw = -\frac{1}{4}\ln|w| + C$

$= -\frac{1}{4}\ln|\cos u| + C = -\frac{1}{4}\ln|\cos(4\theta+9)| + C$

55. $\int \cot x\,dx = \int \frac{\cos x}{\sin x}\,dx$ $\qquad u=\sin x,\quad du=\cos x\,dx$

$= \int \frac{1}{u}\,du = \ln|u| + C = \ln|\sin x| + C$

57. $\int \sec^2(4x+9)\,dx$ $\qquad u=4x+9,\quad du=4dx$

$= \frac{1}{4}\int \sec^2 u\,du = \frac{1}{4}[\tan u + C] = \frac{1}{4}\tan(4x+9) + C$

59. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx$ $\qquad u=\sqrt{x}=x^{\frac{1}{2}},\quad du=\frac{1}{2}x^{-\frac{1}{2}}dx=\frac{1}{2\sqrt{x}}\,dx$

$= 2\int \frac{\sec^2(\sqrt{x})}{2\sqrt{x}}\,dx = 2\int \sec^2 u\,du = 2\tan u + C = \boxed{2\tan\sqrt{x} + C}$

61. $\int \sin 4x \sqrt{\cos 4x + 1}\, dx \qquad u = \cos 4x + 1,\quad du = -\sin 4x \cdot 4\, dx$

$= \frac{-1}{4} \int -4\sin 4x \sqrt{\cos 4x+1}\, dx = \frac{-1}{4}\int u^{\frac{1}{2}}\, du = \frac{-1}{4}\left[\frac{2}{3}u^{3/2} + C\right]$

$= \frac{-\sqrt{(\cos 4x+1)^3}}{6} + C$

63. $\int \sec\theta \tan\theta (\sec\theta - 1)\, d\theta \qquad u = \sec\theta - 1,\quad du = \sec\theta\tan\theta\, d\theta$

$= \int u\, du = \frac{u^2}{2} + C = \frac{1}{2}(\sec\theta - 1)^2 + C$

65. $\int e^{14x-7}\, dx \qquad u = 14x - 7,\quad du = 14\, dx$

$= \frac{1}{14}\int 14 e^{14x-7}\, dx = \frac{1}{14}\int e^u\, du = \frac{1}{14}e^u + C$

$= \frac{e^{14x-7}}{14} + C$

67. $\int \frac{e^x\, dx}{(e^x+1)^4} \qquad u = e^x + 1,\quad du = e^x\, dx$

$= \int u^{-4}\, du = \frac{u^{-3}}{-3} + C = \frac{-1}{3u^3} + C = \frac{-1}{3(e^x+1)^3} + C$

69. $\int \frac{e^t\, dt}{e^{2t} + 2e^t + 1} = \int \frac{e^t\, dt}{(e^t+1)^2} \qquad u = e^t + 1,\quad du = e^t\, dt$

$= \int u^{-2}\, du = \frac{u^{-1}}{-1} + C = \frac{-1}{e^t+1} + C$

71. $\int \frac{(\ln x)^4\, dx}{x} \qquad u = (\ln x),\quad du = \frac{1}{x}\, dx$

$= \int u^4\, du = \frac{u^5}{5} + C = \frac{(\ln x)^5}{5} + C$

73. $\int \frac{\tan(\ln x)}{x}\,dx \qquad u = \ln x,\ du = \frac{1}{x}dx$

$= \int \tan u\,du = \int \frac{\sin u}{\cos u}\,du \qquad w = \cos u,\ dw = -\sin u\,du$

$= -\int \frac{-\sin u}{\cos u}\,du = -\int \frac{1}{w}\,dw = -\ln|w| + C$

$= -\ln|\cos u| + C = \boxed{-\ln|\cos(\ln x)| + C}$

75. $\int \frac{dx}{(1+\sqrt{x})^3} \qquad u = 1+\sqrt{x},\ du = \frac{1}{2\sqrt{x}}\,dx \rightarrow dx = 2\sqrt{x}\,du$
$dx = 2(u-1)\,du$

$= \int \frac{2(u-1)}{u^3}\,du = 2\int (u^{-2} - u^{-3})\,du = 2\left[\frac{u^{-1}}{-1} - \frac{u^{-2}}{-2} + C\right]$

$= \frac{-2}{u} + \frac{1}{u^2} + C = \boxed{\frac{-2}{1+\sqrt{x}} + \frac{1}{(1+\sqrt{x})^2} + C}$

77. $\int \sin x \cos x\,dx \qquad u = \sin x,\ du = \cos x$

$= \int u\,du = \frac{u^2}{2} + C = \boxed{\frac{1}{2}\sin^2 x + C}$

or... $\int \sin x \cos x\,dx \qquad u = \cos x \quad du = -\sin x\,dx$

$= -\int -\sin x \cos x\,dx = -\int u\,du = -\left[\frac{u^2}{2} + C\right]$

$= \boxed{-\frac{1}{2}\cos^2 x + C}$

Since $\int \sin x \cos x\,dx$ could equal either $\frac{1}{2}\sin^2 x + C$ or $-\frac{1}{2}\cos^2 x + C$, we know $\frac{1}{2}\sin^2 x$ and $-\frac{1}{2}\cos^2 x$ only differ by a constant. They have same shape (thus, the same derivative)

Please do NOT think $\frac{1}{2}\sin^2 x$ equals $-\frac{1}{2}\cos^2 x$.
That is a common misconception.

79. If $u = 3x + \pi$ for $\int_0^{\pi} \sin(3x+\pi)\,dx$

new limits would be: $u(0) = 3(0) + \pi = \pi \leftarrow$ lower limit

$u(\pi) = 3\pi + \pi = 4\pi \leftarrow$ upper limit

81. $\int_1^3 (x+2)^3\,dx$ $\quad u = x+2, \quad du = dx$

$u(1) = 3$

$u(3) = 5$

$$= \int_3^5 u^3\,du = \frac{u^4}{4}\Big|_3^5 = \frac{5^4}{4} - \frac{3^4}{4} = \frac{625-81}{4} = 136$$

83. $\frac{1}{2}\int_0^2 \frac{2\,dx}{\sqrt{2x+5}}$ $\quad u = 2x+5, \quad du = 2\,dx$

$u(0) = 5, \quad u(2) = 9$

$$= \frac{1}{2}\int_5^9 \frac{du}{\sqrt{u}} = \frac{1}{2}\int u^{-\frac{1}{2}}\,du = \frac{1}{2}\left[2u^{\frac{1}{2}}\right]_5^9 = \sqrt{u}\Big|_5^9$$

$$= \sqrt{9} - \sqrt{5}$$

85. $\frac{1}{2}\int_0^1 \frac{2x}{(x^2+1)^3}\,dx$ $\quad u = x^2+1, \quad du = 2x\,dx$

$u(0) = 1, \quad u(1) = 2$

$$= \frac{1}{2}\int_1^2 u^{-3}\,du = \frac{1}{2}\left[\frac{u^{-2}}{-2}\right]_1^2 = \frac{-1}{4u^2}\Big|_1^2 = \frac{-1}{16} + \frac{1}{4} = \frac{3}{16}$$

87. $\frac{1}{2}\int_0^4 2x\sqrt{x^2+9}\,dx$ $\quad u = x^2+9, \quad du = 2x\,dx$

$u(0) = 9, \quad u(4) = 25$

$$= \frac{1}{2}\int_9^{25} \sqrt{u}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_9^{25} = \frac{1}{3}u^{3/2}\Big|_9^{25} = \frac{125}{3} - \frac{27}{3} = \frac{98}{3}$$

89. $\int_0^1 (x+1)(x^2+2x)^5\,dx$ $\quad u = x^2+2x, \quad du = (2x+2)\,dx = 2(x+1)\,dx$

$u(0) = 0, \quad u(1) = 3$

$$= \frac{1}{2}\int_0^3 u^5\,du = \frac{1}{2}\left[\frac{u^6}{6}\right]_0^3 = \frac{243}{4}$$

91. $\int_1^e \frac{\ln x}{x}\,dx$ $\qquad u=\ln x,\ du=\frac{1}{x}dx$

$u(1)=\ln(1)=0,\ u(e)=\ln e=1$

$$=\int_0^1 u\,du = \frac{u^2}{2}\Big|_0^1 = \frac{1}{2}$$

93. $\frac{1}{2}\int_0^1 \theta\tan(\theta^2)\,d\theta\,(2)$ $\qquad u=\theta^2,\ du=2\theta\,d\theta$

$u(0)=0,\ u(1)=1$

$$=\frac{1}{2}\int_0^1 \tan u\,du = \frac{1}{2}\int_0^1 \frac{\sin u}{\cos u}\,du$$

$w=\cos u,\ dw=-\sin u\,du$

$w(0)=1,\ w(1)=\cos(1)$

$$=-\frac{1}{2}\int_1^{\cos(1)} \frac{dw}{w} = -\frac{1}{2}\Big[\ln|w|\Big]_1^{\cos(1)}$$

$$=-\frac{1}{2}\Big[\ln(\cos 1)-\ln 1\Big] = -\frac{1}{2}\ln(\cos 1)\approx .3078$$

95. $(-)\int_0^{\frac{\pi}{2}} \cos^3 x\sin x\,dx\,(-1)$ $\qquad u=\cos x,\ du=-\sin x\,dx$

$u(0)=1,\ u(\frac{\pi}{2})=0$

$$=-\int_1^0 u^3\,du = -\left[\frac{u^4}{4}\right]_1^0 = \frac{1}{4}$$

97. $\left(\frac{-1}{2}\right)\int_0^2 r\sqrt{5-\sqrt{5-r^2}}\,dr\,(-2)$ $\qquad u=5-r^2,\ du=-2r\,dr$

some books have a 4 here. I like the 5 better.

$u(0)=5,\ u(2)=1$

$$=-\frac{1}{2}\int_5^1 \sqrt{5-\sqrt{u}}\,du$$

$w=5-\sqrt{u}\qquad dw=\frac{-1}{2\sqrt{u}}\,du$

$w(5)=5-\sqrt{5}\qquad w(1)=4$

$du=-2\sqrt{u}\,dw=-2(5-w)\,dw$

$$=\int_{5-\sqrt{5}}^4 \sqrt{w}\,(5-w)\,dw = \int_{5-\sqrt{5}}^4 (5w^{\frac{1}{2}}-w^{3/2})\,dw$$

$$=\frac{10}{3}w^{3/2}-\frac{2}{5}w^{5/2}\Big|_{5-\sqrt{5}}^4$$

$$=\left(\frac{80}{3}-\frac{64}{5}\right)-(15.3169-5.0802)\approx 3.63$$