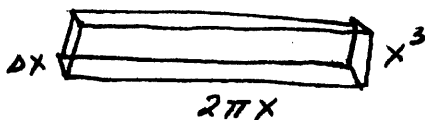
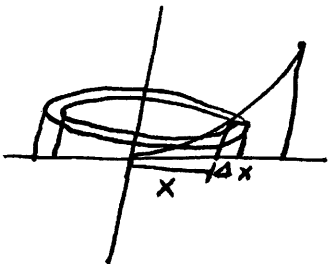


6.4 HW

1. $f(x) = x^3$, $[0, 1]$

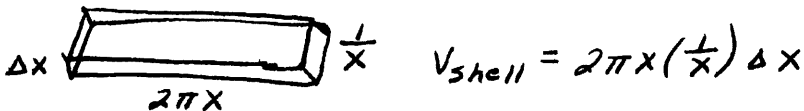


$$V_{\text{shell}} = (2\pi x) x^3 \Delta x$$

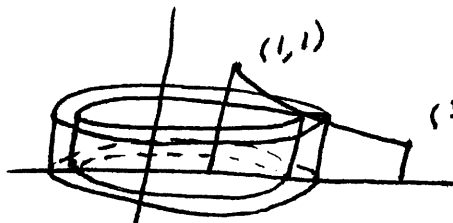


$$V(x) = 2\pi \int_0^1 x^4 dx = 2\pi \left[\frac{x^5}{5} \right]_0^1 = \left(\frac{2\pi}{5} \right)$$

3. $f(x) = x^{-1}$, $[1, 3]$

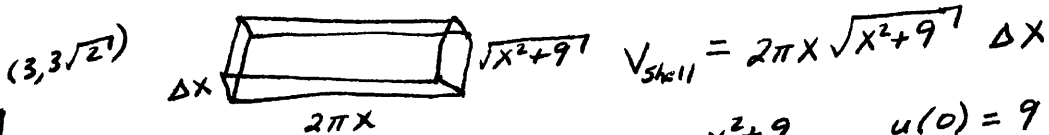
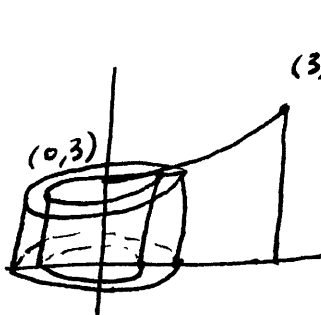


$$V_{\text{shell}} = 2\pi x \left(\frac{1}{x} \right) \Delta x$$



$$V(x) = 2\pi \int_1^3 dx = 2\pi x \Big|_1^3 = 2\pi(2) = (4\pi)$$

5. $f(x) = \sqrt{x^2 + 9}$, $[0, 3]$



$$V_{\text{shell}} = 2\pi x \sqrt{x^2 + 9} \Delta x$$

$$V(x) = 2\pi \int_0^3 x \sqrt{x^2 + 9} dx$$

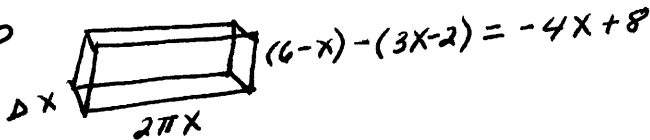
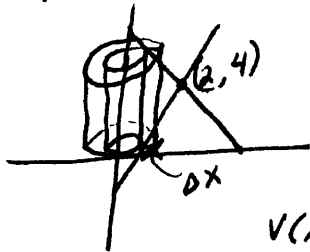
$$u = x^2 + 9 \quad u(0) = 9$$

$$du = 2x dx \quad u(3) = 18$$

$$V(x) = \pi \int_0^3 2x \sqrt{x^2 + 9} dx = \pi \int_9^{18} \sqrt{u} du = \pi \left[\frac{2}{3} u^{3/2} \right]_9^{18}$$

$$V(x) = \pi \left(\frac{2}{3} \right) [18\sqrt{18} - 9\sqrt{9}] = \left(\frac{2\pi}{3} (54\sqrt{2} - 27) \right)$$

7. $y = 3x - 2$, $y = 6 - x$, $x = 0$



$$V_{\text{shell}} = 2\pi x (-4x + 8) \Delta x$$

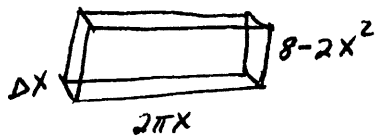
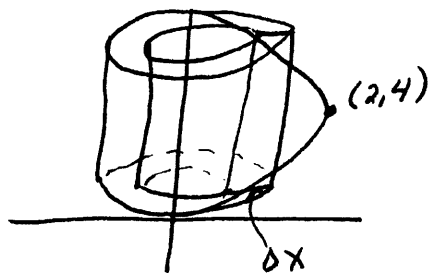
$$V(x) = 2\pi \int_0^2 (-4x^2 + 8x) dx = 2\pi \left[-\frac{4x^3}{3} + \frac{8x^2}{2} \right]_0^2$$

$$V(x) = 2\pi \left(\frac{16}{3} \right) = \frac{32\pi}{3}$$

6.4 HW p.2

9. $y = x^2$, $y = 8 - x^2$, $x = 0$

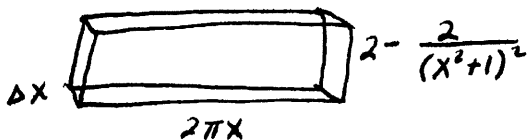
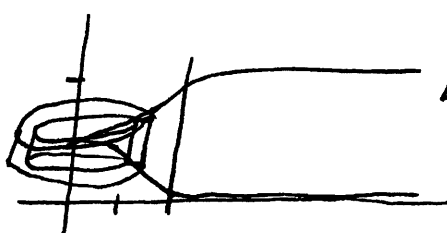
$$\begin{aligned} x^2 &= 8 - x^2 \\ 2x^2 &= 8 \\ x^2 &= 4 \\ x &= 2 \end{aligned}$$



$$V_{shell} = 2\pi x(8 - 2x^2)\Delta x$$

$$V(x) = 2\pi \int_0^2 (8x - 2x^3) dx = 2\pi \left[4x^2 - \frac{1}{2}x^4 \right]_0^2 = 2\pi(8) = \boxed{16\pi}$$

11. $y = \frac{1}{(x^2+1)^2}$, $y = 2 - \frac{1}{(x^2+1)^2}$, $x = 2$



$$V_{shell} = 2\pi x \left(2 - \frac{2}{(x^2+1)^2} \right) \Delta x$$

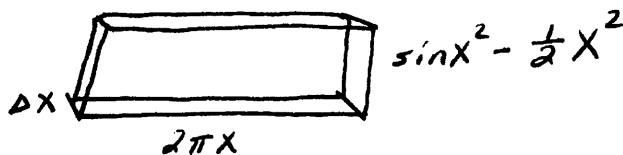
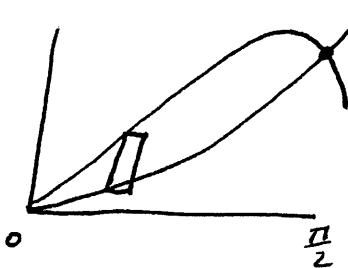
$$V(x) = 2\pi \int_0^2 \left(2x - \frac{2x}{(x^2+1)^2} \right) dx = 2\pi \int_0^2 2x dx + 2\pi \int_0^2 \frac{-2x}{(x^2+1)^2} dx$$

$$\begin{aligned} u &= x^2 + 1 & u(0) &= 1 \\ du &= 2x dx & u(2) &= 5 \end{aligned}$$

$$V = 2\pi [x^2]_0^2 - 2\pi \int_1^5 u^{-2} du$$

$$V = 2\pi(4) - 2\pi \left[\frac{-1}{u} \right]_1^5 = 8\pi - 2\pi \left(1 - \frac{1}{5} \right) = 8\pi - \frac{8\pi}{5} = \boxed{\frac{32\pi}{5}}$$

13. $y = \frac{1}{2}x^2$, $\sin x^2$ $x \geq 0$



$$V_{shell} = 2\pi x (\sin x^2 - \frac{1}{2}x^2) \Delta x$$

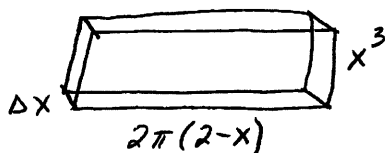
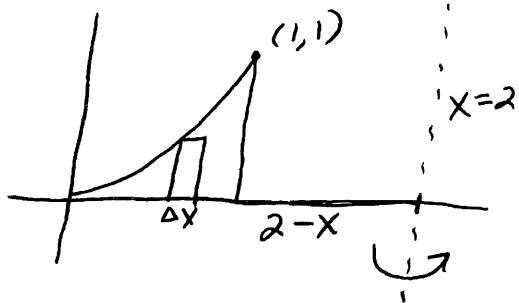
$$V(x) = 2\pi \int_0^{1.377} x \sin x^2 dx - \pi \int_0^{1.377} x^3 dx$$

$$V = \pi \int_0^{1.896} \sin u du - \pi \left[\frac{1}{4}x^4 \right]_0^{1.377} = \pi [-\cos u]_0^{1.896} - \pi [.899]$$

$$V = \pi (.32 + 1) - .899\pi = \boxed{.421\pi}$$

6.4 HW p.3

15. $f(x) = x^3$

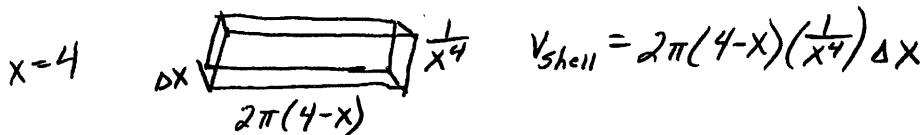
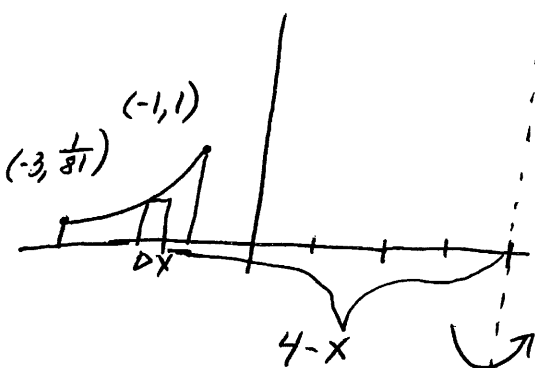


$V_{shell} = 2\pi(2-x)x^3 \Delta x$

$V(x) = 2\pi \int_0^2 (2x^3 - x^4) dx$

$V(x) = 2\pi \left[\frac{x^4}{2} - \frac{x^5}{5} \right]_0^2 = 2\pi \left[\frac{1}{2} - \frac{1}{5} \right] = \frac{3\pi}{5}$

17. $f(x) = x^{-4}$, $[-3, -1]$, about $x=4$

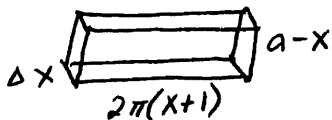
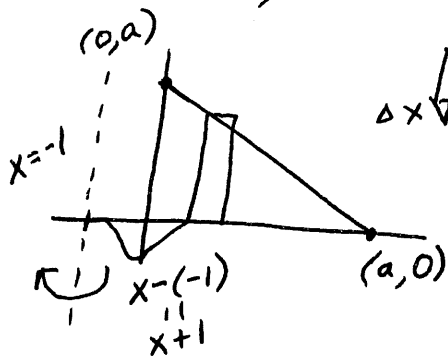


$V(x) = 2\pi \int_{-3}^{-1} (4x^{-4} - x^{-3}) dx$

$V(x) = 2\pi \left[\frac{-4}{3x^3} + \frac{1}{2x^2} \right]_{-3}^{-1} = 2\pi \left(\frac{11}{6} - \frac{17}{162} \right)$

$V = 2\pi \left(\frac{140}{81} \right) = \frac{280\pi}{81}$

19. $f(x) = a-x$, $a > 0$ on $[0, a]$ about $x=-1$



$V_{shell} = 2\pi(x+1)(a-x) \Delta x$

$V(x) = 2\pi \int_0^a (x+1)(a-x) dx$

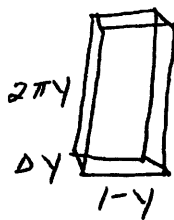
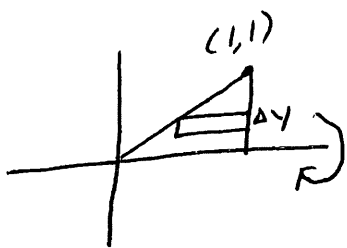
$V(x) = 2\pi \int_0^a (ax - x^2 + a - x) dx$

$V(x) = 2\pi \left[\frac{ax^2}{2} - \frac{x^3}{3} + ax - \frac{x^2}{2} \right]_0^a$

$V = 2\pi \left(\frac{1}{2}a^3 - \frac{1}{3}a^3 + a^2 - \frac{1}{2}a^2 \right) = 2\pi \left(\frac{1}{6}a^3 + \frac{1}{2}a^2 \right) = \frac{\pi a^3}{3} + \pi a^2$

6.4 HW p.4

21. $x=y, y=0, x=1$

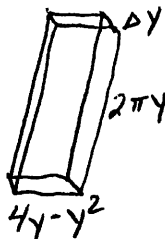
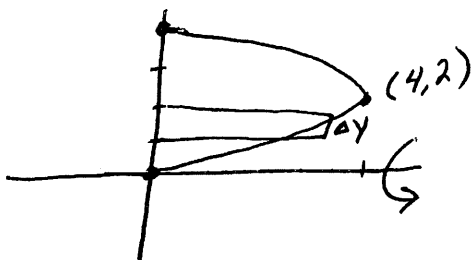


$$V_{shell} = 2\pi y(1-y)\Delta y$$

$$V(y) = 2\pi \int_0^1 (y-y^2) dy = 2\pi \left[\frac{y^2}{2} - \frac{y^3}{3} \right]_0^1$$

$$V(y) = 2\pi \left(\frac{1}{2} - \frac{1}{3} \right) = \frac{\pi}{3}$$

23. $x=y(4-y), x=0$

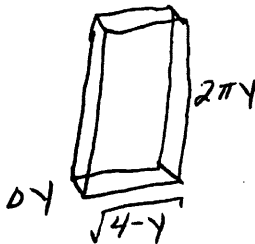
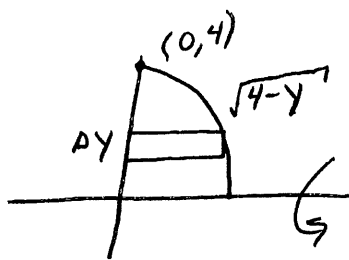


$$V_{shell} = 2\pi y(4y-y^2)\Delta y$$

$$V(y) = 2\pi \int_0^4 (4y^2-y^3) dy$$

$$V(y) = 2\pi \left[\frac{4y^3}{3} - \frac{y^4}{4} \right]_0^4 = 2\pi \left(\frac{64}{3} \right) = \frac{128\pi}{3}$$

25. $y=4-x^2, x=0, y=0$



$$V_{shell} = 2\pi y \sqrt{4-y} \Delta y$$

$$V(y) = 2\pi \int_0^4 y \sqrt{4-y} dy$$

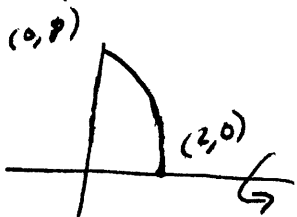
$\rightarrow y=4-u$
 $u=4-y$
 $du=-dy$
 $u(0)=4$
 $u(4)=0$

$$V(u) = 2\pi \int_0^4 (4-u) \sqrt{u} du$$

$$V(u) = 2\pi \int_0^4 (4u^{\frac{1}{2}} - u^{\frac{3}{2}}) du$$

$$V = 2\pi \left[\frac{8}{3} u^{\frac{3}{2}} - \frac{2}{5} u^{\frac{5}{2}} \right]_0^4 = 2\pi \left(\frac{64}{3} - \frac{64}{5} \right) = \frac{256\pi}{15}$$

29. $y=8-x^3, x=[0,2]$



$$V_0 = \pi (8-x^3)^2 \Delta x$$

$$V_0 = \pi (64-16x^3+x^6) \Delta x$$

a.) $V(x) = \pi \int_0^2 (64-16x^3+x^6) dx$

$$= \pi \left[64x - 4x^4 + \frac{x^7}{7} \right]_0^2 = \frac{576\pi}{7}$$

b.) shell \rightarrow

$$V(x) = 2\pi \int_0^2 (8x-x^4) dx$$

$$= 2\pi \left[4x^2 - \frac{x^5}{5} \right]_0^2 = 2\pi \left(\frac{48}{5} \right) = \frac{96\pi}{5}$$