1. $\int x \sin x \, dx$ $\quad u = x \quad dv = \sin x \, dx$
$\quad du = dx \quad v = -\cos x$

$$\int x \sin x \, dx = -x\cos x - \int -\cos x \, dx = -x\cos x + \sin x + C$$

3. $\int (2x+9)e^x \, dx$ $\quad u = 2x+9 \quad dv = e^x dx$
$\quad du = 2dx \quad v = e^x$

$$\int (2x+9)e^x \, dx = (2x+9)e^x - \int 2e^x dx = (2x+9)e^x - 2e^x + C$$
$$= (2x+7)\,e^x + C$$

5. $\int x^3 \ln x \, dx$ $\quad u = \ln x \quad dv = x^3 dx$
$\quad du = \frac{1}{x}dx \quad v = \frac{1}{4}x^4$

$$\int x^3 \ln x \, dx = \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3 dx = \frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$$
$$= \frac{1}{16}x^4(4\ln x - 1) + C$$

7. $\int (4x-3)e^{-x} dx$ $\quad u = 4x-3 \quad dv = e^{-x}dx$
$\quad du = 4dx \quad v = -e^{-x}$

$$\int (4x-3)e^{-x}dx = (3-4x)e^{-x} + 4\int e^{-x}dx = (3-4x)e^{-x} - 4e^{-x} + C$$
$$= e^{-x}(-1-4x) + C$$

9. $\int x e^{5x+2} dx$ $\quad u = x \quad dv = e^{5x+2}dx$
$\quad du = dx \quad v = \frac{1}{5}e^{5x+2}$

$$\int x e^{5x+2} dx = \frac{1}{5}x e^{5x+2} - \frac{1}{5}\int e^{5x+2}dx = \frac{1}{5}x e^{5x+2} - \frac{1}{25}e^{5x+2} + C$$
$$= \frac{1}{25}e^{5x+2}(5x-1) + C$$

11. $\int x \cos 2x \, dx$ $\quad u = x \quad dv = \cos 2x \, dx$
$\quad du = dx \quad v = \frac{1}{2}\sin 2x$

$$\int x \cos 2x \, dx = \frac{1}{2}x\sin 2x - \frac{1}{2}\int \sin 2x \, dx = \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + C$$

13. $\int x^2 \sin x \, dx$ $\quad u = x^2,\ du = 2x\,dx \quad dv = \sin x\,dx,\ v = -\cos x$

$\int x^2 \sin x \, dx = -x^2\cos x + \int 2x\cos x\,dx$ $\quad u = 2x,\ du = 2dx \quad dv = \cos x\,dx,\ v = \sin x$

$\int x^2 \sin x \, dx = -x^2\cos x + 2x\sin x - 2\int \sin x\,dx$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

15. $\int e^{-x}\sin x\,dx$ $\quad u = \sin x,\ du = \cos x\,dx \quad dv = e^{-x}dx,\ v = -e^{-x}$

$\int e^{-x}\sin x\,dx = -e^{-x}\sin x + \int e^{-x}\cos x\,dx$ $\quad u = \cos x,\ du = -\sin x\,dx \quad dv = e^{-x}dx,\ v = -e^{-x}$

$\int e^{-x}\sin x\,dx = -e^{-x}\sin x - e^{-x}\cos x - \int e^{-x}\sin x\,dx$

$2\int e^{-x}\sin x\,dx = -e^{-x}\sin x - e^{-x}\cos x$

$\int e^{-x}\sin x\,dx = \dfrac{-e^{-x}\sin x - e^{-x}\cos x}{2} + C$

17. $\int e^{-5x}\sin x\,dx$ $\quad u = \sin x,\ du = \cos x\,dx \quad dv = e^{-5x}dx,\ v = -\frac{1}{5}e^{-5x}$

$\int e^{-5x}\sin x\,dx = -\frac{1}{5}e^{-5x}\sin x + \frac{1}{5}\int e^{-5x}\cos x\,dx$

$u = \cos x,\ du = -\sin x\,dx \quad dv = e^{-5x}dx,\ v = -\frac{1}{5}e^{-5x}$

$\int e^{-5x}\sin x\,dx = -\frac{1}{5}e^{-5x}\sin x - \frac{1}{25}e^{-5x}\cos x - \frac{1}{25}\int e^{-5x}\sin x\,dx$

$\frac{26}{25}\int e^{-5x}\sin x\,dx = -\frac{1}{5}e^{-5x}\sin x - \frac{1}{25}e^{-5x}\cos x$

$\int e^{-5x}\sin x\,dx = \frac{-5}{26}e^{-5x}\sin x - \frac{1}{26}e^{-5x}\cos x + C$

$= \frac{-1}{26}e^{-5x}(5\sin x + \cos x) + C$

19. $\int x \ln x \, dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{2}x^2$

$\int x \ln x \, dx = \frac{1}{2}x^2 \ln x - \frac{1}{2}\int x\,dx = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$

$= \frac{1}{4}x^2(2\ln x - 1) + C$

21. $\int x^2 \ln x \, dx$

$u = \ln x \qquad dv = x^2 dx$

$du = \frac{1}{x}dx \qquad v = \frac{1}{3}x^3$

$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \frac{1}{3}\int x^2 dx = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$

$= \frac{1}{3}x^3(\ln x - \frac{1}{3}) + C$

23. $\int (\ln x)^2 dx$

$u = (\ln x)^2 \qquad dv = dx$

$du = 2(\ln x)(\frac{1}{x})dx \qquad v = x$

$\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \ln x \, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{1}{x}dx \qquad v = x$

$\int (\ln x)^2 dx = x(\ln x)^2 - 2\left[x \ln x - \int 1\,dx\right] = x(\ln x)^2 - 2x\ln x + 2x + C$

25. $\int x \sec^2 x \, dx$

$u = x \qquad dv = \sec^2 x\,dx$

$du = dx \qquad v = \tan x$

$\to \tan x = \frac{\sin x}{\cos x}$

$\int x \sec^2 x \, dx = x\tan x - \int \tan x\,dx$

$t = \cos x$

$dt = -\sin x\,dx$

$\int x \sec^2 x\,dx = x\tan x + \int \frac{1}{t}dt$

$= x\tan x + \ln|t| + C = x\tan x + \ln|\cos x| + C$

27. $\int \cos^{-1}x \, dx$

$u = \cos^{-1}x \qquad dv = dx$

$du = \frac{-1}{\sqrt{1-x^2}} \qquad v = x$

$t = 1 - x^2$

$dt = -2x\,dx$

$\int \cos^{-1}x\,dx = x\cos^{-1}x + \int \frac{x}{\sqrt{1-x^2}}dx$

$= x\cos^{-1}x - \frac{1}{2}\int t^{-\frac{1}{2}}dt = x\cos^{-1}x - t^{\frac{1}{2}} + C$

$= x\cos^{-1}x - \sqrt{1-x^2} + C$

29. $\int \sec^{-1}x\,dx$

$u = \sec^{-1}x \qquad dv = dx$

$du = \frac{1}{x\sqrt{x^2-1}} \qquad v = x$

$\int \sec^{-1}x\,dx = x\sec^{-1}x - \int \frac{1}{\sqrt{x^2-1}}\,dx$

can't handle this integral directly yet so use #80

#80 $x\sec^{-1}x - \ln\left|x+\sqrt{x^2-1}\right| + C$

31. $\int 3^x \cos x\,dx$

$u = \cos x \qquad dv = 3^x\,dx$

$du = -\sin x\,dx \qquad v = \frac{1}{\ln 3}3^x$

$\int 3^x\cos x\,dx = \frac{3^x}{\ln 3}\cos x + \frac{1}{\ln 3}\int 3^x \sin x\,dx$

$u = \sin x \qquad dv = 3^x\,dx$

$du = \cos x\,dx \qquad v = \frac{1}{\ln 3}3^x$

$\int 3^x\cos x\,dx = \frac{3^x}{\ln 3}\cos x + \frac{1}{\ln 3}\left[\frac{3^x}{\ln 3}\sin x - \frac{1}{\ln 3}\int 3^x\cos x\,dx\right]$

$\int 3^x\cos x\,dx = \frac{3^x}{\ln 3}\cos x + \frac{3^x}{(\ln 3)^2}\sin x - \frac{1}{(\ln 3)^2}\int 3^x\cos x\,dx$

$\left(1+\frac{1}{(\ln 3)^2}\right)\int 3^x\cos x\,dx = \frac{3^x}{\ln 3}\cos x + \frac{3^x}{(\ln 3)^2}\sin x + C$

$\int 3^x\cos x\,dx = \frac{\frac{3^x}{(\ln 3)^2}\left((\ln 3)\cos x + \sin x\right)}{\left(1+\frac{1}{(\ln 3)^2}\right)} + C$

$= \frac{3^x\left((\ln 3)\cos x + \sin x\right)}{(\ln 3)^2+1} + C$

37. $\int e^{\sqrt{x}}\,dx$ $\quad t = x^{\frac{1}{2}} \rightarrow dt = \frac{1}{2}x^{-\frac{1}{2}}dx = \frac{1}{2\sqrt{x}}\,dx$

so $dx = 2\sqrt{x}\,dt = 2t\,dt$

$$\int e^{\sqrt{x}}\,dx = \int e^t(2t\,dt) = 2\int te^t\,dt$$

Now... IBP $\quad u = t \quad dv = e^t dt$

$du = dt \quad v = e^t$

$$2\int te^t dt = 2\left[te^t - \int e^t dt\right] = 2te^t - 2e^t + C$$
$$= 2e^t(t-1) + C$$
$$= 2e^{\sqrt{x}}(\sqrt{x}-1) + C$$

39. $\int x\cos 4x\,dx$ $\quad u = x \quad dv = \cos 4x\,dx$

$du = dx \quad v = \frac{1}{4}\sin 4x$

$$\int x\cos 4x\,dx = \frac{1}{4}x\sin 4x - \frac{1}{4}\int \sin 4x\,dx$$
$$= \frac{1}{4}x\sin 4x + \frac{1}{16}\cos 4x + C$$

41. $\int \frac{x\,dx}{\sqrt{x+1}}$ $\quad t = x+1 \rightarrow x = t-1$

$dt = dx$

$$\int \frac{x\,dx}{\sqrt{x+1}} = \int \frac{t-1}{\sqrt{t}}\,dt = \int \left(t^{\frac{1}{2}} - t^{-\frac{1}{2}}\right)dt = \frac{2}{3}t^{\frac{3}{2}} - 2t^{\frac{1}{2}} + C$$
$$= \frac{2\sqrt{(x+1)^3}}{3} - 2\sqrt{x+1} + C$$

43. $\int \cos x \ln(\sin x)\,dx$ $\quad t = \sin x \rightarrow dt = \cos x\,dx$

$\int \cos x \ln(\sin x)\,dx = \int \ln t\,dt$ $\quad$ Now let $\quad u = \ln t \quad dv = dt$

$du = \frac{1}{t}dt \quad v = t$

$$\int \ln t\,dt = t\ln t - \int dt = t\ln t - t + C$$
$$= (\sin x)\ln(\sin x) - \sin x + C$$

45. $\int \sqrt{x}\, e^{\sqrt{x}}\, dx$

$t = \sqrt{x}$

$dt = \frac{1}{2\sqrt{x}}\, dx \rightarrow dx = 2\sqrt{x}\, dt$

$$\int \sqrt{x}\, e^{\sqrt{x}}\, dx = \int \sqrt{x}\, e^{t}(2\sqrt{x}\, dt) = 2\int x\, e^{t}\, dt \leftarrow x = t^2$$

$$= 2\int t^2 e^t\, dt$$

Now... IBP

$u = t^2 \quad dv = e^t dt$

$du = 2t\, dt \quad v = e^t$

$$2\int t^2 e^t dt = 2\left[t^2 e^t - 2\int t e^t dt\right] \rightarrow \begin{matrix} u = t & dv = e^t dt \\ du = dt & v = e^t \end{matrix}$$

$$= 2\left[t^2 e^t - 2\left[t e^t - \int e^t dt\right]\right]$$

$$= 2t^2 e^t - 4te^t + 4e^t + C$$

so... $\int \sqrt{x}\, e^{\sqrt{x}}\, dx = 2x e^{\sqrt{x}} - 4\sqrt{x}\, e^{\sqrt{x}} + 4e^{\sqrt{x}} + C$

47. $\int \frac{\ln(\ln x)\ln x}{x}\, dx$

$t = \ln x$

$dt = \frac{1}{x} dx$

$$\int \frac{\ln(\ln x)\ln x}{x}\, dx = \int (\ln t)(t)\, dt$$

Now... IBP $\rightarrow$ $u = \ln t \quad dv = t\,dt$

$du = \frac{1}{t} dt \quad v = \frac{t^2}{2}$

$$\int (\ln t)(t)\, dt = \frac{1}{2}t^2 \ln t - \frac{1}{2}\int t\, dt = \frac{1}{2}t^2 \ln t - \frac{1}{4}t^2 + C$$

so... $\int \frac{\ln(\ln x)\ln x}{x}\, dx = \frac{1}{2}(\ln x)^2(\ln(\ln x)) - \frac{1}{4}(\ln x)^2 + C$

$$= \frac{1}{4}(\ln x)^2\left[2\ln(\ln x) - 1\right] + C$$

49. $\int_0^3 x e^{4x}\,dx \Rightarrow$ $\quad u = x \quad dv = e^{4x}dx$
$\quad du = dx \quad v = e^{4x}\left(\frac{1}{4}\right)$

$$\int_0^3 x e^{4x}dx = \left[\frac{1}{4}xe^{4x} - \frac{1}{4}\int e^{4x}dx\right]_0^3 = \frac{1}{4}x e^{4x} - \frac{1}{4}\int e^{4x}dx \Big|_0^3$$

$$= \frac{1}{4}x e^{4x} - \frac{1}{16}e^{4x}\Big|_0^3$$

$$= \left(\frac{3}{4}e^{12} - \frac{1}{16}e^{12}\right) - \left(-\frac{1}{16}\right) = \frac{11}{16}e^{12} + \frac{1}{16} \approx 5.142$$

51. $\int_1^2 x\ln x\,dx$ $\quad u = \ln x \quad dv = x\,dx$
$\quad du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$$\int_1^2 x\ln x\,dx = \left[\frac{1}{2}x^2\ln x - \frac{1}{2}\int x\,dx\right]_1^2 = \frac{1}{2}x^2\ln x - \frac{1}{4}x^2\Big|_1^2$$

$$= (2\ln 2 - 1) - \left(0 - \frac{1}{4}\right) = 2\ln 2 - \frac{3}{4} \approx .636$$

53. $\int_0^{\pi} e^x \sin x\,dx$ $\quad u = \sin x \quad dv = e^x dx$
$\quad du = \cos x\,dx \quad v = e^x$

$$\int e^x\sin x\,dx = \left[e^x\sin x - \int e^x\cos x\,dx\right]$$ $\quad u = \cos x \quad dv = e^x dx$; $\quad du = -\sin x\,dx \quad v = e^x$

$$\int e^x\sin x\,dx = e^x\sin x - \left[e^x\cos x + \int e^x\sin x\,dx\right]$$

$$2\int e^x\sin x\,dx = e^x\sin x - e^x\cos x$$

$$\int e^x\sin x\,dx = \frac{e^x}{2}(\sin x - \cos x)$$

$$\int_0^{\pi} e^x\sin x\,dx = \frac{e^x}{2}(\sin x - \cos x)\Big|_0^{\pi} = \frac{e^{\pi}}{2}(1) - \frac{1}{2}(-1)$$

$$= \frac{e^{\pi}}{2} + \frac{1}{2} = \frac{e^{\pi}+1}{2}$$