5. $\int \sqrt{16-5x^2}\,dx$ $\qquad x = \frac{4}{\sqrt{5}}\sin\theta \rightarrow dx = \frac{4}{\sqrt{5}}\cos\theta\,d\theta$

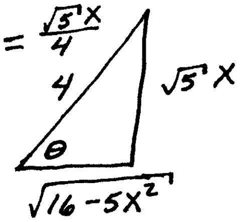
$$16-5x^2 = 16 - 5\left(\tfrac{4}{\sqrt{5}}\sin\theta\right)^2 = 16 - 16\sin^2\theta = 16\cos^2\theta$$

$$= \int \sqrt{16\cos^2\theta}\left(\tfrac{4}{\sqrt{5}}\cos\theta\,d\theta\right) = \frac{16}{\sqrt{5}}\int \cos^2\theta\,d\theta = \frac{8}{\sqrt{5}}\int (1+\cos 2\theta)\,d\theta$$

$\sin 2\theta = 2\sin\theta\cos\theta$

$$= \frac{8}{\sqrt{5}}\left[\theta + \tfrac{1}{2}\sin 2\theta\right] + C = \frac{8\theta}{\sqrt{5}} + \frac{8}{\sqrt{5}}(\sin\theta)(\cos\theta) + C$$

$\sin\theta = \frac{\sqrt{5}\,x}{4}$



$$= \frac{8}{\sqrt{5}}\sin^{-1}\left(\frac{\sqrt{5}\,x}{4}\right) + \frac{8}{\sqrt{5}}\left(\frac{\sqrt{5}\,x}{4}\right)\left(\frac{\sqrt{16-5x^2}}{4}\right) + C$$

$$= \frac{8}{\sqrt{5}}\sin^{-1}\left(\frac{\sqrt{5}\,x}{4}\right) + \frac{x\sqrt{16-5x^2}}{2} + C$$

7. $\int \frac{dx}{x\sqrt{x^2-9}}$ $\qquad x = 3\sec\theta \rightarrow dx = 3\sec\theta\tan\theta\,d\theta$

$$x^2-9 = 9\sec^2\theta - 9 = 9(\sec^2\theta - 1) = 9\tan^2\theta$$

$$= \int \frac{3\sec\theta\tan\theta\,d\theta}{3\sec\theta\,(3\tan\theta)} = \frac{1}{3}\int d\theta = \frac{1}{3}\theta + C$$

$$= \frac{1}{3}\sec^{-1}\left(\frac{x}{3}\right) + C$$

$x = 3\sec\theta$

$\frac{x}{3} = \sec\theta$

$\theta = \sec^{-1}\left(\frac{x}{3}\right)$

9. $\int \frac{dx}{(x^2-4)^{3/2}}$ $\qquad x = 2\sec\theta \rightarrow dx = 2\sec\theta\tan\theta\,d\theta$

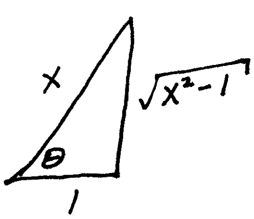
$$x^2-4 = 4\sec^2\theta - 4 = 4\tan^2\theta$$

$$= \int \frac{2\sec\theta\tan\theta\,d\theta}{8\tan^3\theta} = \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{4}\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$u = \sin\theta \rightarrow du = \cos\theta\,d\theta$

$$= \frac{1}{4}\int \frac{1}{u^2}\,du = \frac{1}{4}\left[\frac{u^{-1}}{-1}\right] + C = \frac{-1}{4\sin\theta} + C$$

$$= \frac{-1}{4\left(\frac{\sqrt{x^2-1}}{x}\right)} + C$$



$x = \sec\theta$

$\frac{1}{x} = \cos\theta$

$$= \frac{-x}{4\sqrt{x^2-1}} + C$$

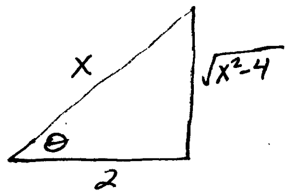
11. $\displaystyle\int \frac{x\,dx}{\sqrt{x^2-4}}$ $\qquad u = x^2-4$, $\quad du = 2x\,dx$

$$= \frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\left[2u^{\frac{1}{2}}\right] + C = \sqrt{x^2-4} + C$$

<u>OR</u>

$\displaystyle\int \frac{x\,dx}{\sqrt{x^2-4}}$ $\qquad x = 2\sec\theta \rightarrow dx = 2\sec\theta\tan\theta\,d\theta$

$x^2-4 = 4\sec^2\theta - 4 = 4\tan^2\theta$



$$= \int \frac{2\sec\theta(2\sec\theta\tan\theta\,d\theta)}{2\tan\theta} = 2\int \sec^2\theta\,d\theta = 2\tan\theta + C$$

$$= 2\left(\frac{\sqrt{x^2-4}}{2}\right) + C = \sqrt{x^2-4} + C$$

ANSWER MATCHES!

13. a.) $\displaystyle\int \frac{x\,dx}{\sqrt{1-x^2}}$ $\qquad u = 1-x^2$, $\quad du = -2x\,dx$

$$= -\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{2}\left[u^{\frac{1}{2}}\cdot 2\right] + C = -\sqrt{1-x^2} + C$$

b.) $\displaystyle\int x^2\sqrt{1-x^2}\,dx$ $\qquad x = \sin\theta$, $\quad dx = \cos\theta\,d\theta$ $\qquad 1-x^2 = 1-\sin^2\theta = \cos^2\theta$

$$= \int (\sin^2\theta)(\cos\theta)\cos\theta\,d\theta = \int \sin^2\theta\cos^2\theta\,d\theta$$

$$= \frac{1}{4}\int (1-\cos^2 2\theta)\,d\theta = \frac{1}{4}\int \sin^2 2\theta\,d\theta$$

$$= \frac{1}{4}\int (1-\cos 2\theta)(1+\cos 2\theta)\,d\theta$$

$$= \frac{1}{8}\int (1-\cos 4\theta)\,d\theta = \frac{1}{8}\theta - \frac{1}{32}\sin 4\theta + C$$

★ $\sin 2\theta = 2\sin\theta\cos\theta$ so

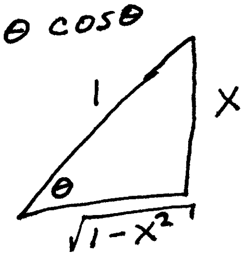
$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sin 4\theta = 2\sin 2\theta\cos 2\theta$

$= 2\left[2\sin\theta\cos\theta\right]\left[\cos^2\theta - \sin^2\theta\right]$

$= 4\sin\theta\cos^3\theta - 4\sin^3\theta\cos\theta$

so $\displaystyle\int x^2\sqrt{1-x^2}\,dx = \frac{1}{8}\theta - \frac{1}{8}\left(\sin\theta\cos^3\theta - \sin^3\theta\cos\theta\right) + C$



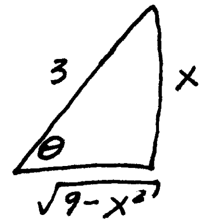
$$= \frac{1}{8}\sin^{-1}x - \frac{1}{8}\left(x\left(\sqrt{1-x^2}\right)^3 - x^3\sqrt{1-x^2}\right) + C$$

Whew! <u>That</u> was fun.
Don't think we were supposed to integrate though. Too tough for part b on #13.

15. $\displaystyle\int \frac{x^2\,dx}{\sqrt{9-x^2}}$ $\quad x = 3\sin\theta \quad dx = 3\cos\theta\, d\theta \quad 9-x^2 = 9-9\sin^2\theta = 9\cos^2\theta$

$$= \int \frac{9\sin^2\theta\,(3\cos\theta\, d\theta)}{3\cos\theta} = 9\int \sin^2\theta\, d\theta = \frac{9}{2}\int (1-\cos 2\theta)\, d\theta$$

$$= \frac{9}{2}\left[\theta - \frac{1}{2}\underbrace{\sin 2\theta}_{2\sin\theta\cos\theta}\right] + C = \frac{9}{2}\theta - \frac{9}{2}\sin\theta\cos\theta + C$$



$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{9}{2}\left(\frac{x}{3}\right)\left(\frac{\sqrt{9-x^2}}{3}\right) + C$$

$$= \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{x\sqrt{9-x^2}}{2} + C$$

17. $\displaystyle\int \frac{dx}{x\sqrt{x^2+16}}$ $\quad x = 4\tan\theta \quad dx = 4\sec^2\theta\, d\theta \quad x^2+16 = 16\tan^2\theta + 16 = 16\sec^2\theta$

$$= \int \frac{4\sec^2\theta\, d\theta}{4\tan\theta\,(4\sec\theta)} = \frac{1}{4}\int \frac{\sec\theta\, d\theta}{\tan\theta}$$

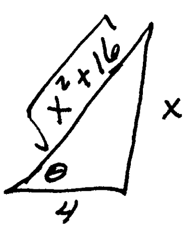
$$\frac{\sec\theta}{\tan\theta} = \frac{1}{\cos\theta}\cdot\frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$$ must use formula 14

$$= \frac{1}{4}\int \csc\theta\, d\theta = \frac{1}{4}\left[\ln\left|\csc\theta - \cot\theta\right|\right] + C$$

$$= \frac{1}{4}\left[\ln\left|\frac{\sqrt{x^2+16}}{4} - \frac{4}{x}\right|\right] + C$$

$x = 4\tan\theta$

$\frac{x}{4} = \tan\theta$



19. $\displaystyle\int \frac{dx}{\sqrt{x^2-9}}$ $\quad x = 3\sec\theta \quad dx = 3\sec\theta\tan\theta\, d\theta \quad x^2-9 = 9\sec^2\theta - 9 = 9\tan^2\theta$

Must use formula 13

$$= \int \frac{3\sec\theta\tan\theta\, d\theta}{3\tan\theta} = \int \sec\theta\, d\theta = \ln\left|\sec\theta + \tan\theta\right| + C$$

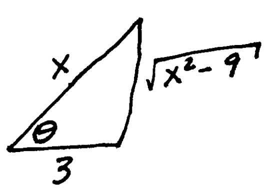
$$= \ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + C$$

$$= \ln\left|\frac{x+\sqrt{x^2-9}}{3}\right| + C$$

$x = 3\sec\theta$

$\frac{x}{3} = \sec\theta$

$\frac{3}{x} = \cos\theta$

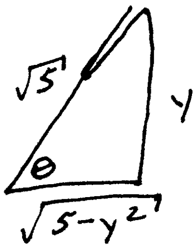


21. $\displaystyle\int \frac{dy}{y^2\sqrt{5-y^2}}$ $\quad y=\sqrt{5}\sin\theta$ $\quad 5-y^2 = 5-5\sin^2\theta = 5\cos^2\theta$

$dy = \sqrt{5}\cos\theta\, d\theta$

$$= \int \frac{\sqrt{5}\cos\theta\, d\theta}{5\sin^2\theta\,(\sqrt{5}\cos\theta)} = \frac{1}{5}\int \frac{1}{\sin^2\theta}\, d\theta = -\frac{1}{5}\int -\csc^2\theta\, d\theta$$

$$= -\frac{1}{5}\left[\cot\theta\right] + C = -\frac{1}{5}\left(\frac{\sqrt{5-y^2}}{y}\right) + C = \frac{-\sqrt{5-y^2}}{5y} + C$$



$\sqrt{5}\sin\theta = y$

$\sin\theta = \frac{y}{\sqrt{5}}$

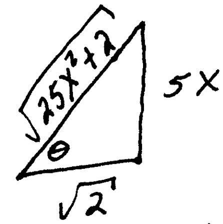
23. $\displaystyle\int \frac{dx}{\sqrt{25x^2+2}} = \int \frac{dx}{5\sqrt{x^2+\frac{2}{25}}}$ $\quad x = \sqrt{\frac{2}{25}}\tan\theta$

$dx = \frac{\sqrt{2}}{5}\sec^2\theta\, d\theta$

$x^2+\frac{2}{25} = \frac{2}{25}\tan^2\theta + \frac{2}{25} = \frac{2}{25}\sec^2\theta$

$$= \int \frac{\frac{\sqrt{2}}{5}\sec^2\theta\, d\theta}{5\left(\frac{\sqrt{2}}{5}\right)\sec\theta} = \frac{1}{5}\int \sec\theta\, d\theta \quad \text{(use formula 13)}$$

$$= \frac{1}{5}\ln\left|\sec\theta + \tan\theta\right| + C$$



$$= \frac{1}{5}\ln\left|\frac{\sqrt{25x^2+2}}{\sqrt{2}} + \frac{5x}{\sqrt{2}}\right| + C$$

$\tan\theta = \frac{5x}{\sqrt{2}}$

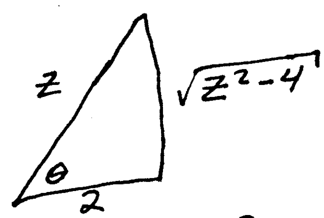
$$= \frac{1}{5}\ln\left|\frac{\sqrt{25x^2+2}+5x}{\sqrt{2}}\right| + C$$

25. $\displaystyle\int \frac{dz}{z^3\sqrt{z^2-4}}$ $\quad z = 2\sec\theta$ $\quad z^2-4 = 4\sec^2\theta - 4 = 4\tan^2\theta$

$dz = 2\sec\theta\tan\theta\, d\theta$

$$= \int \frac{2\sec\theta\tan\theta\, d\theta}{8\sec^3\theta\,(2\tan\theta)} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta} = \frac{1}{8}\int \cos^2\theta\, d\theta = \frac{1}{16}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{16}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C = \frac{1}{16}\left[\theta + \sin\theta\cos\theta\right] + C$$



$$= \frac{1}{16}\left[\cos^{-1}\left(\frac{2}{z}\right) + \frac{\sqrt{z^2-4}}{z}\left(\frac{2}{z}\right)\right] + C$$

$\sec\theta = \frac{z}{2}$

$\cos\theta = \frac{2}{z}$

27. $\int \frac{x^2\,dx}{(6x^2-49)^{1/2}} = \int \frac{x^2\,dx}{\sqrt{6}\sqrt{x^2-\frac{49}{6}}}$

$x = \frac{7}{\sqrt{6}}\sec\theta$

$dx = \frac{7}{\sqrt{6}}\sec\theta\tan\theta\,d\theta$

$x^2 - \frac{49}{6} = \frac{49}{6}\sec^2\theta - \frac{49}{6} = \frac{49}{6}\tan^2\theta$

$= \int \frac{\frac{49}{6}\sec^2\theta\left(\frac{7}{\sqrt{6}}\sec\theta\tan\theta\,d\theta\right)}{\sqrt{6}\left(\frac{7}{\sqrt{6}}\right)\tan\theta}$

$= \frac{49}{6\sqrt{6}}\int \sec^3\theta\,d\theta$ (Use formula #42)

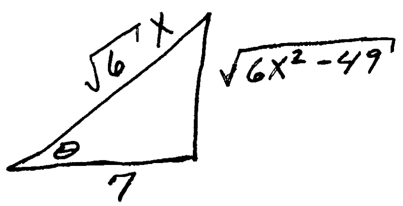
$= \frac{49}{6\sqrt{6}}\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln\left|\sec\theta + \tan\theta\right|\right] + C$

$= \frac{49}{6\sqrt{6}}\left[\frac{1}{2}\left(\frac{\sqrt{6}x}{7}\right)\left(\frac{\sqrt{6x^2-49}}{7}\right) + \frac{1}{2}\ln\left|\frac{\sqrt{6}x}{7} + \frac{\sqrt{6x^2-49}}{7}\right|\right] + C$

$\sec\theta = \frac{\sqrt{6}x}{7}$

$\cos\theta = \frac{7}{\sqrt{6}x}$

$= \frac{49}{12\sqrt{6}}\left[\frac{x\sqrt{6}\sqrt{6x^2-49}}{49} + \ln\left|\frac{x\sqrt{6} + \sqrt{6x^2-49}}{7}\right|\right] + C$



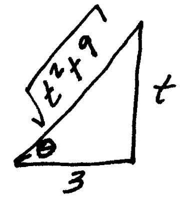
29. $\int \frac{dt}{(t^2+9)^2}$

$t = 3\tan\theta$

$dt = 3\sec^2\theta\,d\theta$

$t^2+9 = 9\tan^2\theta + 9 = 9\sec^2\theta$

$\tan\theta = \frac{t}{3}$



$= \int \frac{3\sec^2\theta\,d\theta}{81\sec^4\theta} =$

$= \frac{1}{27}\int \cos^2\theta\,d\theta = \frac{1}{54}\int (1+\cos 2\theta)\,d\theta = \frac{1}{54}\left[\theta + \frac{1}{2}\sin 2\theta\right] + C$

$= \frac{1}{54}\left[\theta + \sin\theta\cos\theta\right] + C = \frac{1}{54}\left[\tan^{-1}\left(\frac{t}{3}\right) + \frac{t}{\sqrt{t^2+9}}\left(\frac{3}{\sqrt{t^2+9}}\right)\right] + C$

$= \frac{1}{54}\left[\tan^{-1}\left(\frac{t}{3}\right) + \frac{3t}{t^2+9}\right] + C$

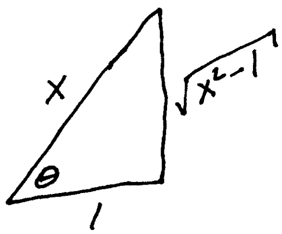
31. $\int \frac{x^2\,dx}{(x^2-1)^{3/2}}$ $\quad x = \sec\theta \quad dx = \sec\theta\tan\theta\,d\theta \quad x^2-1 = \sec^2\theta - 1 = \tan^2\theta$

$$= \int \frac{\sec^2\theta(\sec\theta\tan\theta\,d\theta)}{\tan^3\theta} = \int \frac{\sec^3\theta}{\tan^2\theta}\,d\theta = \int \frac{1}{\cos^3\theta}\cdot\frac{\cos^2\theta}{\sin^2\theta}\,d\theta$$

$$= \int \sec\theta(\csc^2\theta)\,d\theta = \int \sec\theta(1+\cot^2\theta)\,d\theta = \int \sec\theta\,d\theta + \int \sec\theta\cot^2\theta\,d\theta$$

$$= \underset{\text{formula \#13}}{\int \sec\theta\,d\theta} + \underset{\text{we know!}}{\int \cot\theta\csc\theta\,d\theta} = \ln|\sec\theta+\tan\theta| - \csc\theta + C$$

$$= \ln\left|x+\sqrt{x^2-1}\right| - \frac{x}{\sqrt{x^2-1}} + C$$

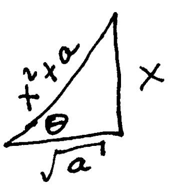


$x = \sec\theta$
$\frac{1}{x} = \cos\theta$

33. $\int \frac{dx}{x^2+a}$ $\quad x = \sqrt{a}\tan\theta \quad dx = \sqrt{a}\sec^2\theta\,d\theta \quad x^2+a = a\tan^2\theta + a = a\sec^2\theta$

$$= \int \frac{\sqrt{a}\sec^2\theta\,d\theta}{a\sec^2\theta} = \frac{1}{\sqrt{a}}\int d\theta = \frac{1}{\sqrt{a}}(\theta) + C$$

$$= \frac{1}{\sqrt{a}}\tan^{-1}\left(\frac{x}{\sqrt{a}}\right) + C$$



$\frac{x}{\sqrt{a}} = \tan\theta$

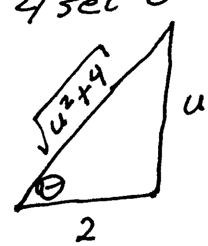
35. $\int \frac{dx}{\sqrt{x^2-4x+8}}$ $\quad (x^2-4x+4)+8-4 = (x-2)^2+4 \quad \left(\frac{-4}{2}\right)^2 = 4$

let $u = x-2 \rightarrow du = dx$

$$= \int \frac{dx}{\sqrt{(x-2)^2+4}} = \int \frac{du}{\sqrt{u^2+4}}$$

$u = 2\tan\theta \quad du = 2\sec^2\theta\,d\theta \quad u^2+4 = 4\tan^2\theta + 4 = 4\sec^2\theta$

$$= \int \frac{2\sec^2\theta\,d\theta}{2\sec\theta} = \int \sec\theta\,d\theta \overset{\#13}{=} \ln|\sec\theta+\tan\theta| + C$$

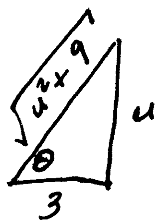


$$= \ln\left|\frac{\sqrt{u^2+4}}{2} + \frac{u}{2}\right| + C = \ln\left|\frac{\sqrt{x^2-4x+8}}{2} + \frac{x-2}{2}\right| + C$$

37. $\displaystyle\int \frac{dx}{\sqrt{x^2+4x+13}}$ $\qquad (x^2+4x+4)+13-4 = (x+2)^2+9$

$$= \int \frac{dx}{\sqrt{(x+2)^2+9}} \qquad \begin{array}{l} u = x+2 \\ du = dx \end{array} \rightarrow \int \frac{du}{\sqrt{u^2+9}} \qquad \begin{array}{l} u = 3\tan\theta \\ du = 3\sec^2\theta\, d\theta \\ u^2+9 = 9\tan^2\theta + 9 \\ \quad = 9\sec^2\theta \end{array}$$

$$= \int \frac{3\sec^2\theta\, d\theta}{3\sec\theta} = \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + C$$



$\frac{u}{3} = \tan\theta$

$$= \ln\left|\frac{\sqrt{u^2+9}}{3} + \frac{u}{3}\right| + C$$

$$= \ln\left|\frac{\sqrt{x^2+4x+13}}{3} + \frac{x+2}{3}\right| + C$$

39. $\displaystyle\int \frac{dx}{\sqrt{6x^2+x}}$ $\qquad 6x^2+x = 6\left(x^2+\frac{1}{6}x+\frac{1}{144}\right) - \frac{6}{144} = 6\left(x+\frac{1}{12}\right)^2 - \frac{1}{24}$

$\left(\frac{\frac{1}{6}}{2}\right)^2 = \frac{1}{144}$

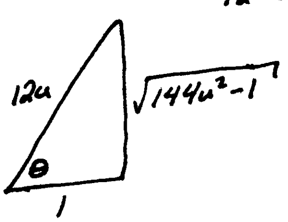
$$= \int \frac{dx}{\sqrt{6(x+\frac{1}{12})^2 - \frac{1}{24}}} = \int \frac{du}{\sqrt{6u^2 - \frac{1}{24}}} = \int \frac{du}{\sqrt{6}\sqrt{u^2 - \frac{1}{144}}} \qquad \begin{array}{l} u = x+\frac{1}{12} \\ du = dx \end{array}$$

$u = \frac{1}{12}\sec\theta \rightarrow u^2 - \frac{1}{144} = \frac{1}{144}\sec^2\theta - \frac{1}{144} = \frac{1}{144}\tan^2\theta$

$du = \frac{1}{12}\sec\theta\tan\theta\, d\theta$

$$= \int \frac{\frac{1}{12}\sec\theta\tan\theta\, d\theta}{\frac{1}{12}\tan\theta} = \int \sec\theta\, d\theta \overset{\#13}{=} \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left|12u + \sqrt{144u^2-1}\right| + C$$



$12u = \sec\theta$

$\frac{1}{12u} = \cos\theta$

$$= \ln\left|12\left(x+\frac{1}{12}\right) + \sqrt{144\left(x+\frac{1}{12}\right)^2 - 1}\right| + C$$

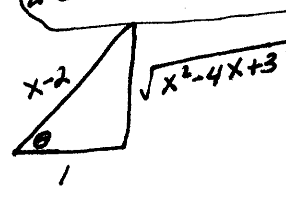
41. $\displaystyle\int \sqrt{x^2-4x+3}\, dx$ $\qquad x^2-4x+3 = (x^2-4x+4)+3-4 = (x-2)^2-1$

$$= \int \sqrt{(x-2)^2-1} \qquad \begin{array}{l} u = x-2 \\ du = dx \end{array} \rightarrow \int \sqrt{u^2-1}\, du \qquad \begin{array}{l} u = \sec\theta \\ du = \sec\theta\tan\theta\, d\theta \\ u^2-1 = \sec^2\theta - 1 = \tan^2\theta \end{array}$$

$$= \int \tan\theta(\sec\theta\tan\theta)\, d\theta = \int \sec\theta\tan^2\theta\, d\theta \rightarrow \begin{array}{l} t = \sec\theta \\ dt = \sec\theta\tan\theta\, d\theta \end{array}$$

Nope, NOT helpful

$$= \int \sec\theta(\sec^2\theta - 1)\, d\theta = \int (\sec^3\theta - \sec\theta)\, d\theta$$



$x-2 = \sec\theta$

$\frac{1}{x-2} = \cos\theta$

$$\overset{\#42,\ \#13}{=} \frac{1}{2}\sec\theta\tan\theta + \frac{3}{2}\ln|\sec\theta + \tan\theta| + C$$

41 cont. $\int \sqrt{x^2-4x+3}\, dx = \frac{1}{2}(x-2)\sqrt{x^2-4x+3} + \frac{3}{2}\ln\left| x-2+\sqrt{x^2-4x+3}\right| + C$

43. $\int \sec^{-1} x\, dx$

$u = \sec^{-1} x \qquad dv = dx$

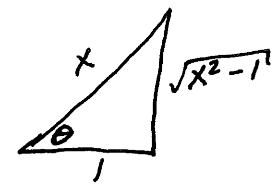
$du = \frac{1}{x\sqrt{x^2-1}} \qquad v = x$

$\int \sec^{-1} x\, dx = x\sec^{-1} x - \int \frac{1}{\sqrt{x^2-1}}\, dx$

let $x = \sec\theta$
$dx = \sec\theta\tan\theta\, d\theta$
$x^2-1 = \sec^2\theta - 1 = \tan^2\theta$

$= x\sec^{-1} x - \int \frac{\sec\theta\tan\theta\, d\theta}{\tan\theta}$

$= x\sec^{-1} x - \int \sec\theta\, d\theta$



$\overset{\#13}{=} x\sec^{-1} x - \ln|\sec\theta + \tan\theta| + C$

$= x\sec^{-1} x - \ln\left| x + \sqrt{x^2-1}\right| + C$

45. $\int \ln(x^2+1)\, dx$

$u = \ln(x^2+1) \qquad dv = dx$

$du = \frac{2x\,dx}{x^2+1} \qquad v = x$

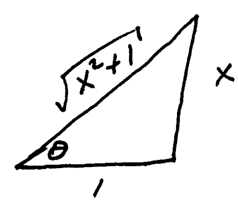
$\int \ln(x^2+1)\, dx = x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}\, dx$

$x = \tan\theta$
$dx = \sec^2\theta\, d\theta$
$x^2+1 = \sec^2\theta$

$= x\ln(x^2+1) - 2\int \frac{\tan^2\theta\sec^2\theta\, d\theta}{\sec^2\theta}$

$= x\ln(x^2+1) - 2\int \tan^2\theta\, d\theta$

$= x\ln(x^2+1) - 2\int (\sec^2\theta - 1)\, d\theta$



$= x\ln(x^2+1) - 2[\tan\theta - \theta] + C$

$= x\ln(x^2+1) - 2x + 2\tan^{-1} x + C$